Oxford Economic Papers 56 (2004), 687–700 **687**
doi:10.1093/oep/gpf056

Relative wage, child labor, and human capital

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This paper analyses child labor and children's human capital formation in response to the changes of the relative wage/productivity between child labor and adult labor. It implies that because children's labor market participation raises the financial resources spent on their education, a small increase in child labor may enhance children's human capital. It also shows that in a poor economy, the laws that punish or partially deter child labor may result in children working more and accumulating less human capital.

1. Introduction

Recently, started with Basu and Van (1998), the issues about child labor have been received increasing attentions in the economic literature and significant contributions are made in this area of research. For example, Basu and Van (1998) analyse the causes of child labor and the policy implications in a model of multiple equilibria in the labor market. Baland and Robinson (2000) study the implications of child labor for welfare, and they show that child labor is socially inefficient when bequests are zero or capital markets are imperfect. Ranjan (2001) argues that credit constraints facing poor households result in excessive child labor and reduce children's human capital. Hazan and Berdugo (2002) explore the dynamic evolution of child labor, fertility, and human capital in the process of development, and they demonstrate the convergence of an economy to a sustained steady-state equilibrium where child labor is abolished.

Most of the existing theoretical literature on child labor, however, has emphasized children's study time as the only input in their human capital accumulation. So, it implies that child labor necessarily reduces children's human capital. This paper extends the existing literature by considering that a child's human capital is determined by the financial resources on her education as well as her time of study. This argument is supported by much empirical evidence. For example, Glewwe *et al.* (2000) and many empirical studies surveyed in their article demonstrate that educational expenditure plays a critical role in children's human capital accumulation in poor countries, and in particular, some of the studies suggest that textbook provision should be given high priority in developing countries. Glewwe *et al.* (2001) find that better nourished children perform significantly better in school.

Horrell *et al.* (2001) show that a household's wealth was a very important factor that determines children's health and human capital in historical times. Indeed, the importance of financial resources on children's human capital formation in poor countries is emphasized by almost every textbook of development economics (e.g. Todaro, 2000).

This paper implies that an important determinant of child labor is the relative wage between child labor and adult labor.[1] Meanwhile, by including both time and money into a human capital formation function, this analysis demonstrates that a rise of child labor productivity may lead to an increase in both child labor and children's human capital. So, in contrast to the conventional wisdom, the model shows that a small increase in child labor may enhance children's human capital since the positive impact of increased financial resources on children's education may outweigh the negative impact of reduced time of study.

The second part of the paper incorporates the 'subsistence constraint' of consumption into the model. The extended model shows that when the subsistence constraint is binding and parental income is low, child labor will decrease with children's relative wage/productivity. This implication is supported by some empirical evidence. For example, Bhalotra (2000) finds that the wage elasticity of child labor is negative for boys when a household's income without child earnings falls below subsistence requirements. Moreover, the analysis implies that the implementation of the laws that punish or partially deter child labor, which results in a reduction of children's wage rate, will both increase child labor and reduce children's human capital when the parental income is below subsistence requirements.

2. The basic model

We consider a small open economy that operates in a perfectly competitive world. The economy is populated by a large number of identical families. Every family has one parent and one child, and the parent is the only decision maker of a household. A parent cares about her family's consumption and her child's human capital.[2] A parent's (i.e., an adult's) utility function is defined as follows

$$V \equiv u(c) + v(h) \tag{1}$$

where c and h denote the household consumption and the child's human capital respectively. We assume

$$u'(c) > 0, u''(c) < 0, v'(h) > 0, v''(h) < 0 \tag{2}$$

[1] This implication is consistent with much empirical evidence (e.g. see the surveys by Basu, 1999, and Fan, 2004). This result is also obtained by Hazan and Berdugo (2002) from a different angle.

[2] Galor and Moav (2002) demonstrate that preferences for the level of offspring's human capital have an evolutionary foundation and they were selected in the process of development.

Every child is endowed with one unit of time, which is divided into two parts: time for work and time for study. We assume that a child's human capital is determined by both the financial resources on her education (x) and her time of study (s). A child's human capital production function is defined as follows

$$h = f(x, s) \tag{3}$$

We assume

$$f_1 > 0, f_{11} < 0, f_2 > 0, f_{22} < 0 \tag{4}$$

We assume that the production function of the consumption good has constant returns to scale and it employs labor and capital. We also assume that the small economy permits unrestricted international lending and borrowing so that its interest rate must be equal to the world interest rate. Thus, its wage rate must be at a fixed level, which is denoted by w.[3] We assume that every adult is endowed with one unit of labor in production. Meanwhile, following Basu and Van (1998), we assume that adults and children are substitutes in production subject to an adult-equivalent scaling, given by γ, where $0 < \gamma < 1$. Thus, an adult's wage and a child's wage rate are w and γw respectively.

We assume that a household cannot borrow against the child's future earnings. Then, a household's budget constraint is

$$c + x = w + (1 - s)\gamma w \tag{5}$$

namely

$$c = w + \gamma w - x - \gamma ws \tag{6}$$

Plugging (3) and (6) into (1), we get

$$V = u(w + \gamma w - x - \gamma ws) + v[f(x, s)] \tag{7}$$

We assume that the optimal solutions are interior, which is the focus of this paper. From (7), the first order conditions are

$$\frac{\partial V}{\partial x} = -u'(w + \gamma w - x - \gamma ws) + v' f_1 = 0 \tag{8}$$

$$\frac{\partial V}{\partial s} = -\gamma w u'(w + \gamma w - x - \gamma ws) + v' f_2 = 0 \tag{9}$$

[3] See Galor and Zeira (1993) and Hazan and Berdugo (2002) for a rigorous derivation. Meanwhile, for simplicity, we assume that the physical capital of the economy completely comes from overseas. So, a household's income is equal to its labor earnings.

Then, we have the following lemma. (The proofs of all of the lemmas and propositions are provided in the Appendix.)

Lemma 1 (1) A child's educational resources increases with the relative wage of child labor (i.e., $(dx/d\gamma) > 0$) if and only if

$$(1-s)u''f_{22} - v''f_1^2 f_2 - \gamma w u'' f_1 - [(1-s)\gamma w u'' + v' f_1] f_{12} > 0 \tag{10}$$

(2) A child's study time decreases with the relative wage of child labor (i.e., $(ds/d\gamma) < 0$) if and only if

$$v'' f_1^3 + u'' f_1 + v' f_1 f_{11} - (1-s)u'' f_{12} + (1-s)\gamma w u'' f_{11} < 0 \tag{11}$$

Note that the first three items in (10) are all positive. So, in particular, (10) will be satisfied if the absolute value of f_{12} is sufficiently small. Meanwhile, note that the first three items in (11) are all negative. Hence, (11) will more likely be satisfied if f_1, which is the marginal product of financial resources in a child's human capital formation function, is larger.

We denote a child's time of working by l_c. As

$$l_c = 1 - s$$

we will have $(dl_c/d\gamma) = -(ds/d\gamma) > 0$ if (11) is satisfied. So, Lemma 1 suggests that a determinant of child labor is the relative wage between child labor and adult labor. We now state the following proposition.

Proposition 1 A marginal increase in child labor (due to a rise of the relative wage of child labor) will enhance a child's human capital (i.e., $(dh/dl_c) > 0$) if (11) is satisfied and $\Omega > 0$, where

$$\Omega \equiv f_1(f_2 f_{11} - f_1 f_{12})v' + [f_1 f_2 - \gamma w f_1^2 + (1-s)(f_1 f_{22} - f_2 f_{12} + \gamma w f_2 f_{11} - \gamma w f_1 f_{12})]u''$$

This proposition implies that a small increase in child labor may enhance children's human capital because children's earnings contribute to household wealth and may raise the expenditure on their education. In fact, the importance of financial constraints on poor children's education not only is demonstrated by the empirical studies that we have discussed in the introduction, but also can simply be reflected by the very low average incomes of many developing countries. For example, even measured by the purchasing power of the local currency, in 1998, the *per capita* incomes of Ethiopia, Nigeria, Bangladesh, and Pakistan were only US \$500, \$820, \$1100, and \$1560 respectively (World Bank, 2000). Clearly, the average incomes are so low that an ordinary family in these countries would often have difficulty in providing enough study materials essential for their

children's education. Therefore, when we include both money and time in a child's human capital production function and when an economy is poor, there is an intriguing relationship between child labor and children's human capital.

Next, we discuss the impacts of government interventions on child labor and children's human capital formation. It should be noted that γ is determined by the implementation of the laws that punish child labor as well as the relative productivity of child labor (e.g. Grootaert and Kanbur, 1995). Suppose a government in an effort to deter the use of child labor, threatens to impose a punitive tax on a firm that employs children, such that the more children the firm employs the higher this punishment. Then, from the firm's point of view this is equivalent to the child having become a less productive input, since with each child employed there is a risk of having to pay a fine. In other words, a partial deterrence measure may be modelled as a lower γ. Then, we have the following proposition.

Proposition 2 The implementation of laws that partially deter child labor can result in children working more and accumulating less human capital if $\Omega > 0$ and

$$v''f_1^3 + u''f_1 + v'f_1f_{11} - (1-s)u''f_{12} + (1-s)\gamma w u''f_{11} > 0 \tag{12}$$

The intuition of this proposition is as follows. As γ rises, there are an income effect and a substitution effect. On one hand, as children's wage rate rises, *ceteris paribus*, a household's wealth will increase and the income effect will cause a child to study more and work less. On the other hand, as the wage rate of child labor rises, a child's opportunity cost of study increases and the substitution effect causes a child to work more. So, the net effect depends on the parameters of the model. Under some circumstances, the income effect dominates the substitution effect so that $(dl_c/d\gamma) < 0$ (i.e., $(ds/d\gamma) > 0$). Meanwhile, although a child works less in response to a rise of γ, the wage rate of children (i.e., γw) increases so that a rise of γ may have little or even positive impact on a child's earnings (i.e., $l_c\gamma w$) and hence the household's wealth under some circumstances. So, the financial resources on children's education may decrease only a little or even increase while children's study time increases in response to an increase in child labor productivity. In this case, we will have $(dh/d\gamma) > 0$. Thus, Proposition 2 implies that the implementation of the laws that punish or partially deter child labor, which results in a reduction of children's wage rate, may both increase child labor and reduce children's human capital.[4]

3. An extended model

In this section, we provide an extension of the basic model by considering that a household faces a subsistence constraint. The importance of the subsistence

[4] Possible negative consequences of partially deterring child labor are also analyzed in Basu and Van (1998) and Basu (2000). This model complements the existing literature from a different angle.

constraint in poor economies is well known. The Brandt Commission drew attention to the fact that '...many hundreds of millions of people in the poorer countries are preoccupied solely with survival and elementary needs...'. (Independent Commission on International Development Issues, 1980, p.49). Todaro (2000, p.363) notes that '...the vast majority (almost 70 per cent) of the world's poorest people ...engaged primarily in subsistence agriculture. Their basic concern is survival...'.

Similar to Davies (1994) and Galor and Weil (2000), we describe a household's subsistence constraint as

$$c \geq \Phi \tag{13}$$

where Φ is the minimum level of consumption for the subsistence of both the parent and the child of a household.[5]

For simplicity and to better illustrate the intuitions of the model, in this section our analyses are based on specific functional forms. We assume that a parent's utility function is as follows

$$V \equiv \ln(c) + \delta \ln(h) \tag{14}$$

where δ is a positive parameter that measures the extent to which parents are altruistic. Meanwhile, a child's human capital production function takes the following Cobb-Douglas form,

$$h = x^{\alpha} s^{\beta} \tag{15}$$

where α and β are both positive coefficients. Plugging (6) and (15) into (14), we get

$$V = \ln(w + \gamma w - x - \gamma w s) + \alpha\delta \ln(x) + \beta\delta \ln(s)$$

Then, we state the following lemma.

Lemma 2 The subsistence constraint is binding if and only if

$$w < \frac{1 + \alpha\delta + \beta\delta}{1 + \gamma}\Phi \tag{16}$$

Note that (16) can be rewritten as

$$w + \gamma w < (1 + \alpha\delta + \beta\delta)\Phi$$

[5] Davies (1994) provides an excellent empirical analysis on this type of subsistence constraint.

which implies that the lower the wage rates of adult workers and child workers, the more likely the subsistence constraint will be binding. Next, we have the following proposition.

Proposition 3 When the subsistence constraint is not binding, we have the following results: (1) Child labor exists if and only if $\gamma > (\beta\delta/(1+\alpha\delta))$. (2) If child labor exists, then we have $(dl_c/d\gamma) > 0$. (3) If child labor exists, then we have

$$\frac{dh}{d\gamma} > (=)(<)0 \quad \text{if } \gamma > (=)(<)\frac{\beta}{\alpha}$$

Similar to Hazan and Berdugo (2002), the first two parts of this proposition show that when the subsistence constraint is not binding, children tend to work more as the relative wage/productivity of child labor increases. This implication is consistent with much historical and contemporary evidence, which shows that child labor productivity and children's labor market participation are closely positively correlated.[6] In an important contribution, Galor and Weil (1996) explain women's increasing labor market participation as a result of the increasing relative wage between women and men. In a similar vein, this proposition suggests that the variation of children's labor market participation in different countries and at different times can be related to the variation in the wage/productivity gap between child labor and adult labor.

The third part of this proposition implies that if the relative productivity of child labor is sufficiently high, an increase in child labor (due to a rise of the relative wage of child labor) will increase children's human capital. From (15), it is easy to verify that the elasticity of human capital with respect to the time devoted to study and the elasticity of human capital with respect to the financial resources on a child's education are β and α respectively. So, if child labor exists, we will have $(dh/d\gamma) > 0$ when the relative wage of child workers is greater than the ratio between these two elasticities.

We now consider the case that subsistence constraint is binding. If the subsistence constraint is binding, then $c = \Phi$. In this case, the Langragian can be written as

$$L = \ln\Phi + \alpha\delta\ln(x) + \beta\delta\ln(s) + \lambda(w + \gamma w - \Phi - x - \gamma ws)$$

So, the first order conditions are

$$\frac{\partial L}{\partial x} = \frac{\alpha\delta}{x} - \lambda = 0 \tag{17}$$

[6] For example, see the empirical studies by Levy (1985), Tuttle (1999), and Bhalotra and Heady (2000).

$$\frac{\partial L}{\partial s} = \frac{\beta\delta}{s} - \lambda\gamma w = 0 \tag{18}$$

Then, we have the following propositions.

Proposition 4 When the subsistence constraint is binding and if child labor exists, we have the following results:

(1) A child's working time is a decreasing function of γ if $w < \Phi$.
(2) A child's working time is an increasing function of γ if $w > \Phi$.

The intuition of Proposition 4 can be explained as follows.[7] When the subsistence constraint is binding, a household's consumption is constant at the level, Φ. Consequently, the parent effectively chooses only her child's time allocation and the resources allocated to the child's human capital formation. Now, to make comparisons, we first consider the case that a parent's wage equals to the consumption constraint (i.e., $w = \Phi$). In this case, the full income of child labor, γw, is devoted solely to the child's human capital. Since the production function of human capital is Cobb-Douglas, full income is divided proportionately between the resources on education and the time spent on study. Meanwhile, since the relative price between the time spent on study and the resources on education is γw, a fixed amount of time is devoted to the formation of human capital. In other words, when $w = \Phi$, the income effect (from an increase in γ) is equal to the substitution effect. Next, we consider the case that the parent's wage is below the subsistence level (i.e., $w < \Phi$). In this case, the income and the substitution effects do not cancel out. It is easy to verify that when $w < \Phi$, as γ increases, the household's full income net of the subsistence level of consumption, $w + \gamma w - \Phi$, increases (in percentage) by more than the cost of child labor, γw. Consequently, the income effect (from an increase in γ) dominates the substitution effect. Thus, when $w < \Phi$, a child's study time increases with γ, and hence her working time decreases with γ. Moreover, by similar logic, we can show that when $w > \Phi$, the income effect will be less than the substitution effect. Thus, when $w > \Phi$, a child's working time increases with γ.

Proposition 5 When the subsistence constraint is binding and when child labor exists, we have the following results

$$\frac{dh}{d\gamma} > (=)(<)0 \quad \text{if } \gamma > (=)(<)\frac{\beta(w-\Phi)}{\alpha w}$$

Similar to Proposition 3, Proposition 5 implies that a small increase in child labor may increase children's human capital when γ is sufficiently high. Meanwhile, holding γ constant, the higher the elasticity of human capital with respect to the

[7] I thank a referee for suggesting this explanation.

financial resources (i.e., α) and the lower the elasticity of human capital with respect to the time of study (i.e., β), the more likely we will have $(dh/d\gamma) > 0$. Also, note that we will have $(\beta(w - \Phi)/\alpha w) < 0$ when $w < \Phi$. So, when adults' wage rate is below the subsistence level, an increase in the wage/productivity of child labor will always enhance children's human capital.

Furthermore, from the proof of Proposition 4 in the Appendix, we can see that child labor always exists when $w < \Phi$. So, Propositions 4 and 5 imply that when $w < \Phi$, children will work less and accumulate more human capital as γ rises. Therefore, we have the following corollary.

Corollary 1 If $w < \Phi$, the implementation of the laws that punish or partially deter child labor will result in that children work more and accumulate less human capital.

Similar to Proposition 2, Corollary 1 shows that government interventions may result in children working more and accumulating less human capital. Meanwhile, Corollary 1 implies that when the subsistence constraint is considered, the results can be obtained under fairly standard assumptions about the utility function and the human capital formation function. Thus, while the laws of child labor usually aim to reduce child labor and enhance children's educational attainment, the model implies that government interventions may sometimes achieve the opposite outcomes. Therefore, this model implies that the government of a developing country may have to implement more sophisticated and more costly measures in order to improve children's welfare effectively. As suggested by some recent empirical studies, these measures may include providing transfers to the poor (either in kind or in cash) on the condition of children attending school on a regular basis (e.g. Ravallion and Wodon, 2000; Skoufias and Parker, 2001).

4. Summary

This paper analyses child labor and children's human capital formation in response to the changes of the relative wage/productivity between child labor and adult labor. By including both time and money into a human capital formation function, we demonstrate that a rise of child labor productivity may lead to an increase in both child labor and children's human capital. So, in contrast to the conventional wisdom, this model shows that a small increase in child labor may not adversely affect children's human capital since the positive impact of increased financial resources on education may outweigh the negative impact of reduced time of study.

Moreover, the model is extended to incorporate the subsistence constraint of consumption. This extension yields several interesting results. First, it shows that when the subsistence constraint is binding and parental income is low, child labor will decrease with children's relative wage/productivity. Second, it implies that if the relative productivity of child labor is sufficiently high, an increase in child labor will increase children's human capital regardless of whether the subsistence constraint is binding. Third, it indicates that when parental income is below subsistence

requirements, the implementation of the laws that partially deter child labor will both increase child labor and reduce children's human capital.

Acknowledgements

I am grateful to three anonymous referees and an editor for constructive comments and suggestions, which improved the quality of the paper significantly. The remaining errors are entirely my own.

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Appendix

Proof of Lemma 1 From (8) and (9), we get

$$-\gamma w f_1 + f_2 = 0 \tag{A.1}$$

Totally differentiating (8) and (A.1) with respect to x, s, and γ, and rearranging, we get

$$\begin{pmatrix} v''f_1^2 + u'' + v'f_{11} & v''f_1f_2 + \gamma wu'' + v'f_{12} \\ f_{12} - \gamma wf_{11} & f_{22} - \gamma wf_{12} \end{pmatrix} \begin{pmatrix} dx \\ ds \end{pmatrix} = \begin{pmatrix} (1-s)wu''d\gamma \\ wf_1 d\gamma \end{pmatrix} \tag{A.2}$$

We define

$$\Delta \equiv \begin{vmatrix} v''f_1^2 + u'' + v'f_{11} & v''f_1f_2 + \gamma wu'' + v'f_{12} \\ f_{12} - \gamma wf_{11} & f_{22} - \gamma wf_{12} \end{vmatrix}$$

Then, we have

$$\begin{aligned} \frac{dx}{d\gamma} &= \frac{1}{\Delta} \begin{vmatrix} (1-s)wu'' & v''f_1f_2 + \gamma wu'' + v'f_{12} \\ wf_1 & f_{22} - \gamma wf_{12} \end{vmatrix} \\ &= \frac{w}{\Delta}\left[(1-s)u''f_{22} - (1-s)\gamma wu''f_{12} - v''f_1^2f_2 - \gamma wu''f_1 - v'f_1f_{12}\right] \end{aligned} \tag{A.3}$$

and

$$\frac{ds}{d\gamma} = \frac{1}{\Delta}\begin{vmatrix} v''f_1^2 + u'' + v'f_{11} & (1-s)wu'' \\ f_{12} - \gamma w f_{11} & wf_1 \end{vmatrix}$$
$$= \frac{w}{\Delta}\left(v''f_1^3 + u''f_1 + v'f_1f_{11} - (1-s)u''f_{12} + (1-s)\gamma wu''f_{11}\right) \tag{A.4}$$

Note that Δ is just the Hessian determinant. So, at the optimum, we must have $\Delta > 0$. Thus, if (10) and (11) are satisfied, from (A.3) and (A.4), we have proved Lemma 1. □

Proof of Proposition 1 From (A.3) and (A.4), we have

$$\frac{dh}{d\gamma} = f_1\frac{dx}{d\gamma} + f_2\frac{ds}{d\gamma} = \frac{w}{\Delta}\Big\{f_1(f_2f_{11} - f_1f_{12})v' + \big[f_1f_2 - \gamma wf_1^2$$
$$+ (1-s)(f_1f_{22} - f_2f_{12} + \gamma wf_2f_{11} - \gamma wf_1f_{12})\big]u''\Big\} = \frac{w\Omega}{\Delta}$$

So, if $\Omega > 0$, we will have $(dh/d\gamma) > 0$. Meanwhile, from Lemma 1, if (11) is satisfied, we will have $(dl_c/d\gamma) > 0$. Thus, we have

$$\frac{dh}{dl_c} = \frac{dh/d\gamma}{dl_c/d\gamma} > 0$$

□

Proof of Proposition 2 From (A.4), we know if (12) is satisfied, we will have $(ds/d\gamma) > 0$, namely $(dl_c/d\gamma) < 0$. So, a partial deterrence of child labor, which reduces γ, will result in an increase in l_c. Meanwhile, as $\Omega > 0$, we have $(dh/d\gamma) > 0$. So, a partial deterrence of child labor, which reduces γ, will result in a decrease in h. □

Proof of Lemma 2 First, suppose that the subsistence constraint is not binding. In this case, from the first order conditions, we get the optimal solutions as

$$c = \frac{w + \gamma w}{1 + \alpha\delta + \beta\delta} \tag{A.5}$$

$$x = \frac{\alpha\delta(w + \gamma w)}{(1 + \alpha\delta + \beta\delta)}, \quad s = \frac{\beta\delta(1+\gamma)}{\gamma(1 + \alpha\delta + \beta\delta)} \tag{A.6}$$

From (A.5), we know that the subsistence constraint is binding if and only if

$$\frac{w + \gamma w}{1 + \alpha\delta + \beta\delta} < \Phi$$

namely

$$w < \frac{1+\alpha\delta+\beta\delta}{1+\gamma}\Phi$$

□

Proof of Proposition 3 (1) From (A.6), we have

$$l_c = 1 - s = \frac{\gamma + \alpha\gamma\delta - \beta\delta}{\gamma(1+\alpha\delta+\beta\delta)} \tag{A.7}$$

So, a child will work (i.e., $l_c > 0$) if and only if

$$\gamma > \frac{\beta\delta}{1+\alpha\delta}$$

(2) From (A.7), we have

$$\frac{dl_c}{d\gamma} = \frac{\beta\delta}{\gamma^2(1+\alpha\delta+\beta\delta)} > 0$$

(3) Plugging (A.6) into (15), we get

$$h = x^\alpha s^\beta = \frac{\alpha^\alpha \beta^\beta \delta^{\alpha+\beta}(w+\gamma w)^\alpha (1+\gamma)^\beta}{\gamma^\beta (1+\alpha\delta+\beta\delta)^{\alpha+\beta}}$$

So

$$\frac{dh}{d\gamma} = \frac{\alpha^\alpha \beta^\beta \delta^{\alpha+\beta}(w+\gamma w)^\alpha (1+\gamma)^{\beta-1}}{\gamma^{\beta+1}(1+\alpha\delta+\beta\delta)^{\alpha+\beta}}(\alpha\gamma - \beta)$$

Thus, $(dh/d\gamma) > (=)(<)\ 0$ if

$$\gamma > (=)(<)\frac{\beta}{\alpha}$$

□

Proof of Proposition 4 From (17), (18), $c = \Phi$, and (5), we can get

$$x = \frac{\alpha}{\alpha+\beta}(w+\gamma w - \Phi), \quad s = \frac{\beta}{\alpha+\beta}\frac{w+\gamma w-\Phi}{\gamma w} \tag{A.8}$$

So, we have

$$l_c = 1 - s = 1 - \frac{\beta}{\alpha+\beta}\frac{w+\gamma w-\Phi}{\gamma w} = \frac{\alpha}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{1}{\gamma}\right)\left(\frac{\Phi}{w} - 1\right) \tag{A.9}$$

So, $l_c > 0$ if $w < \Phi$. And, from (A.9), we have

$$\frac{dl_c}{d\gamma} = \left(\frac{\beta}{\alpha+\beta}\right)\frac{w-\Phi}{\gamma^2 w}$$

Thus, $(dl_c/d\gamma) < 0$ if $w < \Phi$; $(dl_c/d\gamma) > 0$ if $w > \Phi$. □

Proof of Proposition 5 Inserting (A.8) into (15), we have

$$h = x^\alpha s^\beta = \left(\frac{\alpha}{\alpha+\beta}(w+\gamma w-\Phi)\right)^\alpha \left(\frac{\beta}{\alpha+\beta}\frac{w+\gamma w-\Phi}{\gamma w}\right)^\beta$$

$$= \left(\frac{\alpha}{\alpha+\beta}\right)^\alpha \left(\frac{\beta}{\alpha+\beta}\frac{1}{w}\right)^\beta (w+\gamma w-\Phi)^{\alpha+\beta}(\gamma)^{-\beta}$$

So

$$\frac{dh}{d\gamma} = \left(\frac{\alpha}{\alpha+\beta}\right)^\alpha \left(\frac{\beta}{\alpha+\beta}\frac{1}{w}\right)^\beta (w+\gamma w-\Phi)^{\alpha+\beta-1}(\gamma)^{-1-\beta}[(\alpha\gamma-\beta)w+\beta\Phi]$$

Thus, $(dh/d\gamma) > (=)(<)\ 0$ if

$$(\alpha\gamma-\beta)w+\beta\Phi > (=)(<)\ 0$$

namely

$$\gamma > (=)(<)\frac{\beta(w-\Phi)}{\alpha w}$$

□