Human Capital, Study Effort, and Persistent Income Inequality

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Abstract
The paper shows that if an individual’s cost of human capital accumulation depends on his parents’ human capital and there exists a “raw labor” sector of production, individuals with low parental human capital may devote little effort in study and become unskilled workers. Further, if an individual exerts little effort in study, the human capital he accumulated may be even less than his parents’. Consequently, his children will have even lower parental human capital than him and they will therefore also become unskilled. Thus, the model shows that even when education is free, income inequality can persist across generations.

1. Introduction
The persistence of poverty, in many developed as well as developing countries, has been consistently documented and well recognized. In particular, a large body of empirical literature shows that, although the cross-section income distribution may exhibit mean reversion, poverty persists across generations in the lower tails of the income distribution.\(^1\) For example, in his influential book, Wilson (1987) shows that in the United States, while middle-income and some lower-middle-income blacks have moved up in their educational attainment and earnings in the recent few decades, “the truly disadvantaged” low-income blacks have suffered perpetuating poverty.

This important phenomenon motivates much theoretical research. For example, Galor and Zeira (1993) show that when credit markets are imperfect and when the cost of children’s education is sufficiently high, high-income families are better able than poor families to invest in human capital. So, the economy may be characterized by multiple long-run equilibria and income disparities are passed on across generations. Also, several papers (e.g., Benabou, 1993; Fernandez and Rogerson, 1996) study the effects of neighborhood on individuals’ human capital formation. These models show that communities are formed endogenously and, in equilibrium, low-income people and high-income people will live in different communities. Thus, children in a low-income neighborhood will have lower educational attainment, and hence lower earnings because of the lower expenditure on their local schools that they attend. Consequently, poverty persists in low-income neighborhoods.

This paper attempts to complement the existing literature by addressing the proposition that low study effort is another key factor that may cause persistent poverty. The model shows that if an individual’s cost of human capital accumulation depends on his parents’ human capital and there exists a “raw labor” sector of production in which a person’s educational attainment is not an important determinant of his earnings, then individuals with low parental human capital may devote little effort in study and income inequality may persist.

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This paper emphasizes the roles of an individual’s home environment, which is measured by his parental human capital in the model, and his study effort in his human capital formation. The importance of home environment or parental human capital on an individual’s educational attainment has been increasingly recognized in recent literature, but study effort is still largely ignored as an input in human capital accumulation. A notable exception is Glomm and Ravikumar (1992). However, as they assume that human capital accumulated at school is the only factor of production (for every occupation), Glomm and Ravikumar show that parental human capital generally has no effect on children’s study effort at school.

This paper shows that low parental human capital can discourage an individual to exert effort in study, by extending the Glomm–Ravikumar model into a two sector framework: a skilled sector and an unskilled sector. In our model, an individual possesses two factors of production: (1) human capital accumulated at school, and (2) physical labor (or raw labor) that every adult is equally endowed with. We assume that the skilled sector is human-capital-intensive, while the unskilled sector is physical-labor-intensive. This assumption can be illustrated as that a doctor’s or a professor’s productivity is mainly determined by his human capital, while a janitor’s or a porter’s productivity is mainly determined by his physical strength. In particular, this assumption means that a person can take an unskilled job and earn a living with his physical strength, even if he possesses little human capital.

In an overlapping-generations framework and under some reasonable assumptions, this analysis shows that the dynamic interaction between home environment and study effort will make individuals from different families separate into two groups in the long run: a skilled group and an unskilled group. Thus, the economy is characterized by multiple steady states. Each individual in rich dynasties acquires a high level of education and becomes a skilled worker, while each individual in poor dynasties exerts little effort in study and becomes an unskilled worker. The economy’s steady-state equilibrium is path-dependent. It is determined by the initial distribution of human capital of this economy. Thus, the model shows that income may not converge globally to the mean even if education is completely free. Therefore, it complements other models to show that income inequality can persist in the long run.

2. Basic Analytical Framework

Consider individuals who operate in a two-period overlapping-generations world. Every individual belongs to a “family,” where he is a child in his first period and becomes a parent in his second period. Each individual has one parent and one child, so there is no population growth in this economy. In his first period, an individual is endowed with one unit of divisible time, which can be either consumed as leisure or spent in study. We assume that an individual obtains utility from leisure, or equivalently, gets disutility from spending effort in study. Let denote the amount of effort (time) that an individual spends on his study, the corresponding disutility function. We assume that satisfies the following neoclassical property.

Assumption 1

\[ v'(e) > 0, \quad v''(e) > 0, \quad \forall e > 0; \quad v(0) = v'(0) = 0, \lim_{e \to 1} v(e) = \lim_{e \to 1} v'(e) = \infty. \]

Note that we normalize an individual’s minimal amount of effort in study and the corresponding disutility to zero. Let and denote the amount of effort in study, and
the amount of human capital of an individual of generation \( t \), respectively; then the human capital production function is defined as

\[
H_{t+1} = h(H_t, e_{t+1}).
\]  

We assume that the two inputs of the above production function are complementary, and it satisfies the neoclassical property. Then, formally, we have the following.

**Assumption 2**

\[
h_1(H, e) > 0, \ h_2(H, e) > 0, \ h_{11}(H, e) > 0, \ h_{22}(H, e) < 0, \ h_{12}(H, e) < 0, \ \forall \ H, e
\]

\[
\lim_{H \to \infty} h_1(H, e) = \lim_{e \to 1} h_2(H, e) = 0.
\]

We assume that an individual consumes material goods only in his second period, when he earns income from engaging in production. In this period, we assume that an individual obtains utility only from the consumption of material goods. The utility function, which is denoted by \( u() \), is strictly increasing and strictly concave and twice differentiable. For simplicity, we assume that there is no bequest in this economy. Thus, an individual will consume all of his income in his second period.

We assume individuals operate in a small open economy in a one-good world. The good can be produced by two constant-return-to-scale technologies. One is “modern” production technology, which is intensive in human capital (or skilled labor); the other is “traditional” production technology, which is intensive in physical (or unskilled) labor. To capture this property formally in the simplest way, we assume that the only input in the modern (skilled) sector is human capital, while the only input in the traditional (unskilled) sector is physical labor (i.e., raw labor).

Specifically, the production function of the modern technology is described by

\[
Y^m_t = w_s L^s_t,
\]

where \( Y^m_t \) and \( L^s_t \) are the output in this sector and the human capital input of the whole economy at time \( t \), respectively, and \( w_s \) is the marginal productivity of human capital.

The production function of the traditional technology is described by

\[
Y^n_t = w_n L^n_t,
\]

where \( Y^n_t \) and \( L^n_t \) are the output in this sector and the unskilled labor force of the whole economy at time \( t \), respectively, and \( w_n \) is the marginal productivity of physical labor.

The markets for both human capital and physical labor are perfectly competitive. So, the wage rate per unit of human capital is \( w_s \), while the wage rate per unit of physical labor is \( w_n \). These two production technologies are available to every individual. However, since an individual can perform tasks only in one of the two sectors at a time, he may choose to work either as a skilled or an unskilled worker, but not both. Every individual is assumed to be endowed with one unit of physical labor (strength) in his second period (regardless of the level of his education attainment). Thus, if an individual acquires \( H \) amount of human capital in his first period, then his income would be \( w_s H \) if he chooses to be a skilled worker, and \( w_n \) if he chooses to be an unskilled worker.

### 3. Parental Human Capital and Occupational Choice

This model abstracts from the consideration of uncertainty. So, in a world of perfect foresight, an individual’s occupational choice is effectively made in his first period. A
rational individual will choose the occupation that can yield him higher lifetime utility. So, firstly, we will investigate an individual’s optimal choice of the amount of study effort and hence his maximum utility by being either skilled or unskilled.

If an individual (of generation \( t + 1 \)) chooses to be unskilled, his income is certain at \( w_n \), and his intertemporal utility function is

\[
u(w_n) - \nu(e_{t+1}).
\]  

(4)

The constraint is

\[
0 \leq e_{t+1} \leq 1.
\]  

(5)

Since an individual gets disutility from spending effort in study, obviously, the optimal solution to (4) is to set \( e_{t+1} = 0 \). Thus, we have the following lemma.

**Lemma 1.** If an individual chooses to be an unskilled worker, he will spend zero amount of effort in study.

If an individual chooses to be a skilled worker, his intertemporal utility function is

\[
U \equiv u[w_h(H_t, e_{t+1})] - \nu(e_{t+1}).
\]  

(6)

The constraint is also (5). So the first-order condition is

\[
w_u[w_h(H_t, e_{t+1})]h_2(H_t, e_{t+1}) - \nu'(e_{t+1}) = 0.
\]  

(7)

It should be noted that the corner solutions \( e_{t+1} = 0 \) and \( e_{t+1} = 1 \) are excluded by \( \nu'(0) = 0 \) and \( \lim_{e \rightarrow 1} \nu'(e) = \infty \) (Assumption 1), respectively.

Now, we have the following lemma (the proofs of lemmas, theorems, and corollaries are all provided in the Appendix).

**Lemma 2.** If an individual chooses to be a skilled worker, given the amount of his parental human capital and under Assumptions 1 and 2, his optimal choice of the amount of effort in study exists and is unique.

Let the optimal amount of study effort for an individual who chooses to be skilled be \( e_{t+1}^* \). By Lemma 2, \( e_{t+1}^* \) is a function of \( H_t \). So we can define it as

\[
e_{t+1}^* = e(H_t).
\]

Totally differentiating (7) with respect to \( e_{t+1} \) and \( H_t \), we get

\[
\frac{de_{t+1}}{dH_t} = \frac{w_u^2u''h_1h_2 + w_u'u'h_1}{v'' - w_u^2u''h_2^2 - w_u'u'h_{22}}.
\]  

(8)

The denominator of the right-hand side of (8) is clearly positive. The first item of the numerator is negative, which can be regarded as the “income effect” when divided by the denominator; the second item of the numerator is positive, which can be regarded as the “substitution effect” when divided by the denominator. In Glomm and Ravikumar (1992), for example, the “income effect” is exactly offset by the “substitution effect,” as implied by the assumed log-linear utility function of income and leisure. So, the net effect is zero in their model; that is, parental human capital has no effect on children’s study effort at school. Thus, as will be shown, the disincentive effect in
study may exist only for children with very low parental human capital and choosing to be unskilled. However, no matter what the net effect is, we have the following.

**Theorem 1.** *If an individual chooses to be a skilled worker, then, under Assumptions 1 and 2, we have*

\[
\frac{dH_{t+1}}{dH_t} > 0.
\]

The intuition of this theorem can be explained as follows. In Figure 1, \(l \equiv 1 - e\); that is, \(l\) is the amount of leisure that an individual takes. Then, an individual’s “budget constraint” can be described by

\[
c_{t+1} = w_s H_{t+1} = w_s h(H_t, 1 - l_{t+1}).
\]

Clearly, \(c_{t+1}\) decreases as \(l_{t+1}\) increases. So, the “budget curve” is downward-sloping. Meanwhile, as \(h(H_t, 1 - l_{t+1})\) is an increasing function of \(H_t\), the “budget curve” moves up as \(H_t\) increases. Because \(c_{t+1}\) is a normal good as implied by the utility function, \(u(c_{t+1}) - v(1 - l_{t+1})\), the pure income effect resulting from the increase of \(H_t\) will increase \(c_{t+1}\), and hence \(H_{t+1}\). Thus, \(H_{t+1}\) increases as \(H_t\) increases.

This theorem indicates that an individual with higher parental human capital will accumulate more human capital and hence get higher income if he chooses to be skilled. Thus, this theorem complements the previous studies (e.g., Becker and Tomes, 1979; Loury, 1981), by showing that intergenerational earnings are positively correlated even if education is completely free.

Now we define

\[
G(H_t, w_s, w_n) \equiv u(w_s h(H_t, e_{t+1}^*) - v(e_{t+1}^*) - u(w_n)
= u[w_s h(H_t, e(H_t))] - v(e(H_t)) - u(w_n).
\]

![Figure 1](image-url)
Clearly, $G$ measures the difference of an individual’s intertemporal utility of being skilled and that of being unskilled. So an individual will choose to be skilled if and only if $G \geq 0$.

The following lemma discusses the relationship between $G$ and its variables.

**Lemma 3.** $G$ is a strictly increasing function of $H_t$ and $w_s$, and is a strictly decreasing function of $w_n$.

Now we add a technical assumption that serves as a sufficient condition of the next theorem.

**Assumption 3**

There exists a large number $N$, such that $h(N, 0) > w_n/w_s$.

This assumption means that if an individual’s parental human capital is sufficiently high, he would be able to accumulate enough human capital to earn more than an unskilled worker’s wage even if he just spent the (normalized) minimal amount of effort in study. Based on the above description, now we have the following theorem.

**Theorem 2.** Under Assumptions 1–3, there exists a critical value $H^c$, such that an individual will choose to be skilled if and only if his parental human capital is greater or equal to $H^c$.

This theorem establishes the relationship between an individual’s parental human capital and his occupational choice. The higher his parental human capital is, the more likely an individual will choose to be skilled. The intuition is that individuals with high parental human capital have a comparative advantage in accumulating human capital, so individuals with higher parental human capital are more likely to choose to be skilled. Because an individual with low parental human capital is inefficient in accumulating human capital, and because he has to sacrifice leisure time to study hard, he may find it optimal for him to exert little effort in study at school and become an unskilled worker in the future. So, the model shows that a bad home environment may discourage an individual to accumulate human capital even if education is completely free.

Now we will show that under two more assumptions below, individuals will be segmented into two groups of dynasties in the long run.

**Assumption 4**

$$h(H_t, 0) < H_t, \forall H_t (H_t \neq 0); \quad h(0, 0) = 0.$$  

In this model, we normalize an individual’s minimal level of human capital to zero. The above assumption means that no matter what kind of family background an individual has, if he does not study at all his human capital will be less than his parents’. If an individual’s parental human capital is zero, and he does not study at all, his human capital will also be at the minimal level.

**Assumption 5**

$$h(H^c, e(H^c)) > H^c.$$  

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This assumption means that when an individual’s parental human capital is just at the threshold level that he chooses to be skilled, he will accumulate more human capital than that threshold level. Because a skilled worker’s wage is higher than an unskilled worker’s, in particular, \( w_s h(H_e, e(H_e)) \geq w_u \), namely, \( h(H_e, e(H_e)) \geq w_u/w_s \). Thus, the above assumption is weaker than the assumption \( w_u/w_s > H_c \), which means that a skilled worker’s human capital is always above the critical value of human capital \( H_c \). Corollary 3 in the next section will imply that this assumption rules out some unrealistic implications.

Finally, one may point out that if \( e(H_e) = 0 \), then Assumptions 4 and 5 are in contradiction. However, because \( v'(0) = 0 \), it is easy to see from (7) that we always have \( e(H_e) > 0 \).

**Corollary 1.** Under Assumptions 1–5, there exist two groups of dynasties in the economy, where the members of one group will always be skilled, while the members of the other group will always be unskilled.

The essence of this corollary is that it predicts a strong intergenerational correlation in occupational choices. The intuition is that if an individual has a comparative disadvantage in study and becoming skilled, this comparative disadvantage can be reinforced by his little effort in study and will be passed on to his future generations; on the other hand, if an individual has a comparative advantage in accumulating human capital, he will study hard to be skilled. Consequently, his children will benefit from high parental human capital in accumulating enough human capital, and the comparative advantage in study will remain for his future generations.

However, it should be noted that the model implies there is perfect intergenerational transmission of occupational choices only because it abstracts from some other considerations. For example, some random factors, such as individuals’ innate abilities or market luck, may significantly affect an individual’s human capital formation and occupational choice. So, if some individuals with low parental human capital are endowed with high innate abilities, or if some individuals with high parental human capital are endowed with low innate abilities, we will observe more intergenerational mobility. However, even if we take these factors into account, the essence of this corollary will remain that the children whose parents are skilled are (much) more likely to be skilled than those whose parents are unskilled.

This prediction is consistent with some empirical evidence (e.g., Jencks, 1979; Brittain, 1977). For example, based on his eleven panel studies on American men, Jencks (1979, p. 214) shows that “All aspects of family background explained about 48% of the variance in mature men’s occupational statuses (and) the most important single measured background characteristic affecting a son’s occupational status is his father’s occupational status.”

There are two further comments on this model. Firstly, although parental human capital plays a critical role in an individual’s human capital formation and consequently his occupational choice, explicit altruism is assumed away from the model. However, it is not difficult to verify that we can obtain qualitatively the same results when altruism is considered. To see this, suppose we extend the model by assuming that one obtains utility from his human capital itself, for its externality effect on his child’s human capital formation. If we assume this new utility function is concave, from (6), the combination of \( u(w_s H_{it}) \) and the new utility function will have the same qualitative property as \( u() \) with respect to its variable \( H_{it} \). So, from the proofs in the Appendix, it is clear that this consideration will change the decisions of the individuals.
only at the margin of the threshold level, and hence will not change any of our results qualitatively.

Secondly, if an individual has both a skilled parent and an unskilled parent, his learning efficiency will be a function of some sort of (convex) combination of both of his parents’ human capital. However, there is much evidence on the positive assortative mating by education. For example, in the United States, college-educated men are 15 times as likely to marry college-educated women as are men who never completed high school (Becker, 1991). Thus, as a first-order approximation, we might as well ignore this complication that an individual may have both a skilled parent and an unskilled parent.

In this model, we see that an individual’s parental human capital fully determines his occupational choice. Now, let \( D_0 \) be the distributional function of the initial (parental) human capital (at time 0) in this economy. Since each individual has one child and one parent, the total number of the workforce in this economy is constant over time, and we denote it by \( L \). Then, by Corollary 1 and its proof, the number of unskilled labor in this economy at any time \( t \) is

\[
L_t^u = \int_0^{H_c} dD_0(H_t)L = D_0(H^c)L
\]  

(9)

and the number of skilled labor is

\[
L_t^s = \int_{H^c}^{\infty} dD_0(H_t)L = (1 - D_0(H^c))L.
\]  

(10)

Thus, when the initial distribution of parental human capital neither below \( H_c \) nor above \( H^c \) is trivial, individuals will be segmented into two groups of dynasties in the long run.

4. Long-Run Equilibria

In this section, I will analyze the long-run equilibria of the evolution of an economy’s human capital distribution and hence its income distribution. The dynamics of individuals’ human capital of a dynasty is in steady state (at time \( t \)) if

\[
H_t = H_{t+1} = H_{t+2} = \cdots.
\]

From (1), it is easy to see that in steady state, we must also have

\[
e_t = e_{t+1} = e_{t+2} = \cdots.
\]

The following two theorems characterize the properties of the steady states of individuals’ human capital of both groups of dynasties that I described above.

**Theorem 3.** Under Assumptions 1 and 4, the unique and stable steady state of individuals’ human capital of the group of the dynasties who choose to be unskilled is zero.

The intuition behind this theorem is that because an individual’s human capital does not increase his productivity and hence his income when he chooses to be unskilled, individuals of a dynasty who choose to be unskilled will not choose to put effort into study and invest in human capital. So, the individuals’ human capital of the dynasty will become smaller and smaller over time until it reaches the minimal level. This theorem indicates that some individuals whose parents are unskilled may not choose
to be skilled even if the skilled-intensive sector experiences growth (i.e., $w_s$ rises), since the parental human capital of unskilled dynasties becomes increasingly smaller.

**Theorem 4.** Under Assumptions 1–3 and 5, for the group of dynasties who choose to be skilled, (a) there always exists at least one locally stable steady state of individuals’ human capital; (b) there exists a unique steady state of individuals’ human capital if the following inequality is satisfied:

$$
(1 - h_1(H_t, e^*_t))h_{22}(H_t, e^*_t) + h_2(H_t, e^*_t)h_{12}(H_t, e^*_t) \leq 0, \forall H_t (H_t \geq H^*).
$$  

(11)

(c) If inequality (11) is satisfied, then

$$
\frac{dH_{t+1}}{dH_t} < 1.
$$

When $h_1, h_2,$ and $h_{12}$ are small enough such that (11) can be satisfied, individuals’ human capital and hence income will converge within the group of the dynasties who choose to be skilled. In the short run, although the child from a richer family will still be richer (by Theorem 1), his human capital (hence earnings) will be closer to the mean than his parents’, and therefore income will converge across generations within the skilled dynasties. This case is illustrated in Figure 2.

If (11) is not satisfied, there may be multiple equilibria of individuals’ human capital within the skilled dynasties, which implies that income inequality may persist even within the skilled dynasties. However, it should be noted that (11) is only a sufficient, but not necessary, condition for the uniqueness of equilibrium for skilled dynasties. Consider the following example, which illustrates several theorems and assumptions of the paper. Suppose that an individual’s utility function is

$$
\ln I_t + \gamma \ln(1 - e_t),
$$

Figure 2.
where $I_i$ is the individual’s income. Meanwhile, we consider a human capital production function that takes the following Cobb–Douglas form:

$$H_{i+1} = n(H_i, e_{i+1}) = AH_i^\alpha e^{1-\alpha}_{i+1},$$

where $A$ and $\alpha$ are positive coefficients, and $\alpha < 1$. If an individual of generation $t+1$ chooses to be skilled, then his objective function is

$$\ln w_i A H_i^\alpha e^{1-\alpha}_{i+1} + \gamma \ln(1 - e_{i+1})$$
$$= \ln w_i A H_i^\alpha + (1 - \alpha) \ln e_{i+1} + \gamma \ln(1 - e_{i+1}).$$

So, his optimal choice of study effort can be calculated as

$$e^{*}_{i+1} = \frac{1 - \alpha}{1 - \alpha + \gamma}.$$

Thus, an individual will choose to be skilled if and only if

$$\ln w_i A H_i^\alpha + (1 - \alpha) \ln e_{i+1} + \gamma \ln(1 - e_{i+1}) \geq \ln w_n,$$

namely

$$H_i \geq \frac{(1 - \alpha + \gamma)^{1 - \alpha + \gamma}}{(1 - \alpha)\gamma^\alpha} \left(\frac{w_n}{Aw_i}\right)^{\frac{1}{\alpha}}.$$

So, Assumption 5 is satisfied if and only if

$$A \left[\frac{(1 - \alpha + \gamma)^{1 - \alpha + \gamma}}{(1 - \alpha)\gamma^\alpha} \left(\frac{w_n}{Aw_i}\right)^{\frac{1}{\alpha}}\right]^{\frac{1}{\alpha}} - (1 - \alpha) \ln e_{i+1} + \gamma \ln(1 - e_{i+1}) > \frac{(1 - \alpha + \gamma)^{1 - \alpha + \gamma}}{(1 - \alpha)\gamma^\alpha} \left(\frac{w_n}{Aw_i}\right)^{\frac{1}{\alpha}};$$

that is

$$\frac{Aw^{1-\alpha}_{i+1}}{w^{1-\alpha}_{i+1}} > \frac{(1 - \alpha + \gamma)^{1 - \alpha + \gamma}}{(1 - \alpha)\gamma^\alpha}.$$

Clearly, this condition will be satisfied if $A$ or $w_i$ is large enough or $w_n$ is small enough.

If and only if the above condition is satisfied, the nontrivial steady state exists. By definition, the steady states can be calculated as follows:

$$H^* = A (H^*)^\alpha \left(\frac{1 - \alpha}{1 - \alpha + \gamma}\right)^{1 - \alpha}.$$

The trivial steady state obviously exists, and the nontrivial steady state is

$$H^* = A^{\frac{1}{1-\alpha}} \frac{1 - \alpha}{1 - \alpha + \gamma}.$$

To check the (local) stability at the nontrivial steady state, notice that

$$\frac{dH_{i+1}}{dH_i} = \alpha A H_i^{\alpha-1} e^{1-\alpha}_{i+1} = \alpha A \left(A^{\frac{1}{1-\alpha}} \frac{1 - \alpha}{1 - \alpha + \gamma}\right)^{\alpha-1} \left(\frac{1 - \alpha}{1 - \alpha + \gamma}\right)^{1 - \alpha} = \alpha.$$
Because $0 < \alpha < 1$, $dH_{t+1}/dH_t < 1$. So, the nontrivial steady state is stable.

However, in this example, the left-hand side of (11) can be derived as

$$AH^{\alpha-1}\left(\frac{1-\alpha}{1-\alpha+\gamma}\right)^{1-\alpha} - 1 \alpha(1-\alpha)AH^{\alpha}\left(\frac{1-\alpha}{1-\alpha+\gamma}\right)^{-1-\alpha}.$$ 

Clearly, when $A$ is sufficiently large, we can assign some numbers to the parameters and variables so that the above item is positive. So, in this case, (11) is not satisfied. Thus, this example shows that (11) is only a sufficient, but not necessary, condition for the uniqueness of equilibrium for skilled dynasties.

Finally, we observe that in the example the optimal amount of study effort for those who choose to be skilled is independent of parental human capital. In this case, the following corollary shows that the nontrivial steady state is unique.

**Corollary 2.** Under Assumptions 1–3 and 5, if the optimal amount of study effort for those who choose to be skilled is independent of parental human capital, namely

$$\frac{de^*(H_i)}{dH_i} = 0,$$

then there exists a unique steady state of individuals’ human capital for the group of dynasties who choose to be skilled.

Whether the nontrivial steady state is unique or not, we have provided a mechanism showing that if an economy’s initial human capital distribution is sufficiently unequal, its income distribution will not converge to the mean globally even if education is completely free. For example, consider the case that the steady state of individuals’ human capital of the group of skilled dynasties is unique, and it is denoted by $H^*$. In this case, a skilled individual’s income will converge to $w_sH^*$ in the long run. Thus, the above two theorems indicate that the long-run wage inequality between these two group of dynasties is

$$w_sH^* - w_n.$$

Therefore, if the initial human capital distribution is sufficiently unequal, income disparity of an economy may persist even if every individual in the economy can get access to the same amount of educational resources.

The crucial point of this paper is that low study effort, which results in low level of human capital and poverty, comes from an individual’s comparative disadvantage in accumulating human capital. This paper assumes that this comparative disadvantage stems from low parental human capital. Clearly, if we consider the fact that one who has lower parental human capital generally lives in a bad neighborhood and attends a low-quality school, his comparative disadvantage in accumulating human capital will be reinforced, and so will all of the results of this paper. In other words, the basic argument of the paper can be extended to reinforce the results in some previous studies.

Finally, as a comment to Theorem 4, we have the following corollary.

**Corollary 3.** If $h(H^*, e(H^*)) < H$ and if (11) is satisfied, then, under Assumptions 1–3, the only steady state of individuals’ human capital in this economy is zero.

This corollary implies that Assumption 5 is necessary to rule out the unrealistic implication that all individuals will be unskilled in the long run.
In this model, since education is completely free, parents’ earnings per se do not affect their children’s earnings. However, because parents’ human capital affects their children’s learning efficiency at school, which in turn affects their children’s human capital formation, and because an individual’s earnings are associated with his human capital, intergenerational earnings are correlated. Therefore, this model, together with some other models emphasizing the externality effect of community human capital or ethnic human capital (e.g., Benabou, 1993; Borjas, 1992), implies that income inequality may persist even if education is completely public (i.e., free).

Meanwhile, this study indicates that the timing of educational subsidy matters. Specifically, it implies that the government should try to provide good education for every member of the society early. In particular, the government should subsidize the education of the students who have good performance in school work but have financial difficulty in attending school. Otherwise, when an economy’s human capital distribution becomes unequal (resulting from unequal initial income distribution), this model implies that it will be much more difficult for the government to reduce poverty.

Finally, from (9) and (10), we can get the long-run per capita GNP as

\[ D_0(H^c)w_n + (1 - D_0(H^c))w_n H^b. \] (12)

Therefore, the initial distribution of human capital determines the long-run wellbeing of the individuals in this economy. An economy with a more equal distribution of initial human capital will tend to have a higher level of living standard in the steady state.

5. Conclusion

This paper addresses the proposition that low study effort is a key factor that may cause persistent poverty. In an overlapping-generations framework, the model shows that if an individual’s cost of human capital accumulation depends on his parents’ human capital and there exists a “raw labor” sector of production in which a person’s educational attainment is not an important determinant of his earnings, then individuals with low parental human capital may devote little effort in study and income inequality may persist.

Because an individual with low parental human capital is inefficient in accumulating human capital, and because he has to sacrifice leisure time to study hard, he may find it optimal to exert little effort in study at school and become an unskilled worker in the future. So, low parental human capital may discourage an individual to accumulate human capital even if education is completely free. What is worse, this disincentive effect in study may have an intergenerational effect and generates a “poverty trap.” Because an individual who plans to be an unskilled worker exerts little effort in study, the human capital he accumulated may be even less than his parents’. Consequently, his children will have even lower parental human capital than him and they will therefore also choose to be unskilled. This process will be repeated from one generation to the next, and all of his offspring will choose to be unskilled. In other words, if an individual has a comparative disadvantage in study and becoming skilled, this comparative disadvantage can be reinforced by his little effort in study and will be passed on to his future generations. Thus, the model shows that even if education is free, income inequality can persist across generations.
Appendix

Proof of Lemma 2

Because \( U \) is a continuous function with respect to \( e_{t+1} \), and because \( e_{t+1} \in [0, 1] \), which is a compact set, the optimal solution must exist. And because

\[
\frac{d^2 U}{d e_{t+1}^2} = w_s u'' h_2^2 + w_s u' h_{22} - \nu'' < 0,
\]

the optimal solution must be unique.

Proof of Theorem 1

Totally differentiating (1), rearranging, and plugging (8), we get

\[
\frac{d H_{t+1}}{d H_t} = h_1 + h_2 \frac{de_{t+1}}{d H_t} = \frac{h_1 \nu'' - w_s u' h_{22} + w_s u' h_{22}}{\nu'' - w_s^2 u'' h_2^2 - w_s u' h_{22}} > 0. \tag{A1}
\]

Proof of Lemma 3

By the envelope theorem

\[
\frac{\partial G}{\partial H_t} = u'(w_s H_{t+1})w_s h_1 > 0
\]

and

\[
\frac{\partial G}{\partial w_s} = H_{t+1}u'(w_s H_{t+1}) > 0, \quad \frac{\partial G}{\partial w_n} = -u'(w_n) < 0.
\]

Proof of Theorem 2

Noticing Assumption 3, we have

\[
G(H_t = N) = u[w_s h(N, e(N))] - \nu(e(N)) - u(w_n) > u\left(\frac{w_n}{w_s}\right) - 0 - u(w_n) = 0.
\]

Case 1: \( G(H_t = 0) < 0 \). In this case, by the continuity of \( G() \), there must be an \( H^c \in (0, N) \), such that \( G(H_t = H^c) = 0 \). Besides, by Lemma 3, we know \( \partial G/\partial H_t > 0 \). So \( H^c \) is unique. Case 2: \( G(H_t = 0) \geq 0 \). In this case, because \( \partial G/\partial H_t > 0 \) (Lemma 3), we have \( G(H_t = 0) > 0 \) for all \( H_t \). Thus, every individual will choose to be skilled. So, in this case, \( H^c = 0 \).

Proof of Corollary 1

If an individual’s parental human capital \( H_t \) satisfies \( H_t \geq H^c \), by Theorem 2, this individual will choose to be a skilled worker. By Theorem 1 and Assumption 5, we have

\[
H_{t+1} = h(H_t, e(H_t)) \geq h(H^c, e(H^c)) > H^c.
\]
So, again by Theorem 1, this individual’s child will also choose to be a skilled worker. This process will be repeated from one generation to the next, and all of this individual’s offspring will choose to be skilled. On the other hand, if an individual’s parental human capital $H_t$ is less than $H^c$, namely, $H_t < H^c$, by Theorem 1, this individual will choose to be an unskilled worker. In this case, by Lemma 1, he will spend zero (i.e., minimal) amount of effort in study. Then, by Assumption 4, his human capital will be

$$H_{t+1} = h(H_t, 0) < H_t < H^c.$$ 

Thus, his child will also choose to be an unskilled worker and will spend zero effort in study. This process will be repeated from one generation to the next, and all of this individual’s offspring will choose to be unskilled.

**Proof of Theorem 3**

By Lemma 1, we see that when an individual chooses to be unskilled, the amount of time that he will spend on study is zero. Then, from Assumption 4, $H = 0$ (and hence $e = 0$) is obviously the unique steady state. And at $H = 0$, by Assumption 4

$$\frac{dH_{t+1}}{dH_t} = \lim_{H \to 0} \frac{h(H, 0) - 0}{H - 0} < 1.$$ 

So it is stable.

**Proof of Theorem 4**

(1) First, at $H_t = H^c$, by Assumption 5, we have $H_{t+1} > H_t$. Second, the concavity and the Inada condition of $h(H, e)$ with respect to $H$ (for any given $e$) can guarantee that there exists some $H^0$ (which is sufficiently large), such that $H^0 > h(H^0, 1)$. Thus, when $H_t = H^0$, we have

$$H_{t+1} = h(H^0, e(H^0)) \leq h(H^0, 1) < H^0.$$ 

Therefore, by continuity, there exists $H^c < H_t < H^0$, such that $H_{t+1} = H_t$. So we must have at least one steady state.

Also, Theorem 1 indicates that $dH_{t+1}/dH_t > 0$, namely, the locus of the dynamics of individuals’ human capital, in the diagram where $H_t$ and $H_{t+1}$ represent horizontal axis and vertical axis respectively, is strictly upward-sloping. Thus, at the first intersection between this locus and the 45-degree line, we must have $dH_{t+1}/dH_t < 1$, since this locus must cross the 45-degree line from above at this point. Thus, this steady state is stable.

(2) To prove the second and the third result, we only need to show that when (11) is satisfied, we will have $dH_{t+1}/dH_t < 1$, $\forall H_t$. Noticing that $dH_{t+1}/dH_t > 0$ (Theorem 1), we see that from (A1), $dH_{t+1}/dH_t < 1$ is equivalent to

$$h_1 v'' - w_2 u' h_{12} + w_2 u' h_{22} < v'' - w_2^2 u'' h_2^2 - w_2 u' h_{22}.$$ 

On rearranging we get

$$(1 - h_1)v'' - w_2^2 u'' h_2^2 - w_2 u'[(1 - h_1)h_{22} + h_2 h_{12}] > 0.$$  \hspace{1cm} (A2)
So when (11) is satisfied, (A2) will be satisfied. Then

\[ 0 < \frac{dH_{t+1}}{dH_t} < 1. \]

Thus, the nontrivial steady state is unique.

**Proof of Corollary 2**

By Assumption 5, at the steady state with the smallest \( H \), the locus of the dynamics of individuals’ human capital of a dynasty goes through the 45-degree line from above. Meanwhile, from Theorem 1, we know \( h(H, e^*(H)) \) increases with \( H \). Thus, at the steady state with the smallest \( H \), we must have \( 0 < \frac{dH_{t+1}}{dH_t} < 1 \). Also, if \( de^*(H)/dH_t = 0 \), then \( d^2H_{t+1}/dH_t^2 = h_{11} < 0 \). Thus, the slope of the locus of the dynamics of individuals’ human capital after the steady state with the smallest \( H \) must be less than one and positive. Therefore, the locus of the dynamics of individuals’ human capital of a dynasty will never go through the 45-degree line again after the steady state with the smallest \( H \) (otherwise, we would have \( dH_{t+1}/dH_t \geq 1 \)). So, the nontrivial steady state is unique.

**Proof of Corollary 3**

Consider a diagram in which \( H_t \) and \( H_{t+1} \) represent horizontal axis, and vertical axis, respectively. If \( H_t < H^* \), similar to the situation in Theorem 3 or Corollary 1, the dynamics of individuals’ human capital of this dynasty will reach the steady state \( H = 0 \). On the other hand, if \( H_t \geq H^* \), because \( |dH_{t+1}/dH_t| < 1 \) when (11) is satisfied (by Theorem 4), the locus of the dynamics of individuals’ human capital of a dynasty will never go through the 45-degree line; otherwise, we would have \( |dH_{t+1}/dH_t| \geq 1 \). Thus, the steady state above \( H^* \) cannot exist. So the only steady state in this economy would be at \( H = 0 \).

**References**


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Notes

2. For some recent literature, see Becker et al. (1990), Galor and Tsiddon (1997a,b), and Hanushek (1996).
3. Simply speaking, we may call these two factors of production as mental labor and physical labor respectively, as in Galor and Weil (1996).
4. It should be noted that it is possible that an individual stays at school but spends no effort in study. There is much anecdotal evidence that many high-school graduates in New York city are illiterate although they have stayed in school for 12 years, which indicates that they spent little effort in study at school.
5. The “income effect” and “substitution effect” in this context are interpreted as follows. As an individual’s parental human capital increases, his efficiency of study will increase. Consequently, his income will increase (holding other variables constant) and his opportunity cost of taking leisure will also increase. An individual will consume more leisure (study less hard) as his income increases (“income effect”), while he will consume less leisure (study harder) as the (opportunity) cost of taking leisure increases (“substitution effect”).
6. Galor and Tsiddon (1997a) analyzes an interesting model in which both parental human capital and individual ability determine individual earnings.