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Oxford Economic Papers 56 (2004), 735–743 **735**
doi:10.1093/oep/gpf065

Process and product R&D by a multiproduct monopolist

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It is shown that the claim in Lambertini that a multiproduct monopolist's incentive for process R&D declines with the number of products it offers is incorrect. This incentive is in fact an increasing function of the number of products in his model. I further extend the model of Lambertini to show that whether or not process R&D incentive and the number of varieties are positively related depends on the degree of scope economies in process R&D. Product innovation promotes process R&D if the degree of such scope economies is high (as in Lambertini), and discourages it if the degree of scope economies is low.

1. Introduction

In a recent article, Lambertini (2003) examines the product and process R&D decisions of a multiproduct monopolist and compares them with that of the social planner. The author derives two main results: (i) the monopolist's incentive for cost-reducing R&D is inversely related to the number of products it produces (Lemma 1); and (ii) monopoly causes efficiency distortions in both product and process R&D. In this note, I show that the first result of Lambertini (2003) is invalid: the incentive for process R&D is in fact positively related to the number of varieties in his model. I correct an error in his derivations and use an example with simple R&D function to illustrate the positive relation. Moreover, I extend the model of Lambertini, which is equivalent to the case with perfect scope economies in process R&D, to the general setting where the scope economies parameter can vary from 0 to 1. By doing this, I show that if the scope economy is absent (or its degree is low), the monopolist's incentive for process R&D is indeed negatively related to the number of varieties.

2. Model of Lambertini

A monopoly firm offers n products, the demand system for which is given by

$$p_i = \alpha - q_i - \gamma \sum_{j \neq i} q_j, \quad i = 1, 2, \ldots, n$$

The products are substitutes if $\gamma \in (0, 1)$ and complements if $\gamma \in (-1, 0)$. Total costs of the firm are given by

$$C(Q, k) = c(k) \sum_{i=1}^{n} q_i + \xi k^2 + \theta n F \tag{1}$$

where $Q \equiv (q_1, q_2, \ldots, q_n)$ and $F > 0$ is the fixed cost of introducing a product; θ is the scope economies parameter in production with $\theta \in [0, 1]$ for $n > 1$ and $\theta = 1$ for $n = 1$. Variable k represents the level of process R&D. Note that k pertains to the (common) marginal cost of production for each product, $c(k)$. (In the second part of this paper, I consider product-specific process R&D.) It is assumed that $c' < 0, c'' \geq 0$ and there is no uncertainty in R&D.

3. Optimal R&D portfolio under monopoly

The monopolist decides how many products to offer as well as the amount of process R&D. The basic trade-off, captured nicely by the above demand and cost functions, lies in the need to balance the benefit of economies of scope in production with limiting product 'cannibalization'.

For given $\{n, k\}$ the optimal quantity mix Q is the solution to

$$\text{Maximize } \Pi_M \equiv \sum_{i=1}^{n} \left[\alpha - q_i - \gamma \sum_{j \neq i} q_j - c(k) \right] q_i - \theta n F - \xi k^2$$

Deriving the first order conditions and imposing symmetry yields

$$q_i = q^* = \frac{\alpha - c(k)}{2[1 + \gamma(n-1)]} \tag{2}$$

The monopoly equilibrium profits are

$$\Pi_M(n, k) = \frac{n[\alpha - c(k)]^2}{4[1 + \gamma(n-1)]} - \theta n F - \xi k^2 \tag{3}$$

The optimal R&D portfolio for the monopolist, $\{n, k\}$, is thus the solution to

$$\text{Maximize } \Pi_M(n, k)$$

As shown by Lambertini (Appendix 1), the first-order conditions are

$$n = \frac{\gamma - 1}{\gamma} + \frac{[\alpha - c(k)]\sqrt{\theta F(1-\gamma)}}{2\theta F \gamma}$$

and

$$c'(k) = -\frac{4\xi k[1+\gamma(n-1)]}{n[\alpha - c(k)]} \tag{4}$$

respectively.

To examine how the optimal process R&D investment depends on n, Lambertini differentiates the first order condition of k with respect to n. In doing so, however, he held k constant and ended up with the following expression (eq. (17), p.569)

$$\frac{\partial c'(k)}{\partial n} = \frac{4\xi k(1-\gamma)}{n^2[\alpha - c(k)]} \geq 0$$

Based on this, the following result was reported[1]

Lemma 1 (Lambertini, 2003) The monopolist's incentive towards process innovation is decreasing in the number of products supplied in equilibrium.

The correct approach is that one should also take into account the effect of a change in n on k. To see this, rewrite (4) as

$$g(k, n) \equiv c'(k) + \frac{4\xi k[1+\gamma(n-1)]}{n[\alpha - c(k)]} = 0$$

which implicitly defines the optimal $k(n)$ for given n. Thus, $g(k(n), n) = 0$ for all n. Differentiating this equation with respect to n, we get

$$\frac{\partial g}{\partial k}\frac{dk(n)}{dn} + \frac{\partial g}{\partial n} = 0$$

By the second order condition of k, we have $\partial g/\partial k < 0$ at the optimum. Since the term $\partial g/\partial n$ is positive, as the author has shown, it must be that

$$\frac{dk(n)}{dn} > 0$$

Of course, the same conclusion is obtained if one explicitly differentiates (4) with respect to n keeping in mind that $k = k(n)$. In fact, it is straightforward to show (after rearranging terms and utilizing the first-order conditions of the

[1] Lambertini also derives the optimal R&D portfolio for the social planner and compares it with that of the monopoly. A nice result he obtained is that monopoly causes distortions in both product and process R&D dimensions. In particular, it is shown that compared with social optimum the monopolist's product range is smaller when products are substitutes and is the same when products are complements. It is also shown that for any given number of products, the monopoly operates at a higher marginal cost than does the social planner.

monopolist) that

$$\frac{dk(n)}{dn} = \frac{-(1-\gamma)4\xi k/n}{n(c')^2 - n(\alpha - c(k))c'' - 4\xi[1+(n-1)\gamma]}$$

By the second order condition for k, the denominator in the above expression is negative. Therefore, $dk(n)/dn > 0$.

3.1 An example: $C(k) = \bar{c} - k$

Assume that the process R&D function takes a simple form: $c(k) = \bar{c} - k$. In this case, the first order condition (4) becomes

$$1 = \frac{4\xi k[1+\gamma(n-1)]}{n[\alpha - \bar{c} + k]}$$

which gives us

$$k(n) = \frac{n(\alpha - \bar{c})}{4\xi[1+\gamma(n-1)] - n} = \frac{(\alpha - \bar{c})}{4\xi[((1-\gamma)/n) + \gamma] - 1}$$

Obviously, $k(n)$ is an increasing function of n in contrast to Lemma 1 of Lambertini.

Therefore, the correct version of Lemma 1 in Lambertini (2003) should be as follows.

Lemma 1′ The monopolist's incentive towards process innovation is increasing in the number of products supplied, i.e., $dk(n)/dn > 0$.

3.2. General intuition

The above result should not come as a surprise. In general, a firm's incentive to conduct process R&D is positively related to the level of output it produces. This is so because cost-reducing R&D lowers the unit cost of production and, thus, the larger the output produced, the greater the cost savings. In this model, the monopolist' aggregate output for given $\{n, k\}$ is (by eq. (2))

$$Q = nq = \frac{n[\alpha - c(k)]}{2[1+\gamma(n-1)]} = \frac{\alpha - c(k)}{2[((1-\gamma)/n) + r]}$$

which obviously increases with n.

Alternatively, one can look at the marginal benefit of investment in process R&D. By (3), it is easy to see that the marginal benefit of process R&D for the monopolist, as given below, is greater for larger n.

$$\frac{\partial \Pi_M}{\partial k} = \frac{[\alpha - c(k)](-c')}{2[(1-\gamma)/n + r]}$$

All the above arguments apply equally to the social planner.

4. Economies of scope in process R&D

One distinct feature of Lambertini (2003), as can be seen from eq. (1), is that process R&D applies to the common marginal cost of all n products offered by the monopolist: a \$$k$ investment in process R&D by the firm reduces the marginal cost of producing each and every one of the products by the same amount. Put differently, process R&D investment in his model applies to the 'common costs' of the multiproduct monopolist. Below, I show that if one considers process R&D that lowers the 'attributable costs', that is, if process R&D is product specific, then the incentive for process innovation may either increase or decrease with the number of products being offered, depending on the degree of scope economies in process R&D.

Suppose that instead of (1) the monopolist's overall costs are given by

$$C(Q, k) = \sum_{i=1}^{n} c(k_i)q_i + \theta nF + \sum_{i=1}^{n} \xi k_i^2$$

where $c(k_i)$ is the marginal cost for producing product i, and k_i is the amount of process R&D aimed specifically at lowering $c(k_i)$, $c' < 0$ and $c'' \geq 0$. To simplify, I further assume that $c(k_i) = \bar{c} - k_i$.

More generally, one could consider the case that $c(\cdot)$ is a function of

$$K_i \equiv k_i + \beta \sum_{j \neq i} k_j$$

where $\beta \in [0,1]$ measures the degree of scope economies in process R&D — process R&D investment for a given product of the multiproduct firm may benefit R&D projects it conducts for other (related) products. With this specification, K_i represents the 'effective R&D investments' for product i.[2]

Within this general setting, the model of Lambertini (2003) is the special case that $\beta = 1$ (perfect scope economies in R&D). The point of this section is to show

[2] In the literature on process R&D by single-product firms (see, e.g., d'Aspremont and Jacquemin, 1988), a firm's R&D investment may spillover to its competitors in the industry so the 'effective R&D' there can be represented by $X_i = x_i + \lambda \sum_{j \neq i} x_j$, where x_j are R&D investments of rival firms. In that literature, R&D spillovers, measured by λ, stem from the presence of various spillover channels such as labor turnover, reverse engineering, and other forms of imitation. In my setting here for a multiproduct monopolist, the parameter of scope economies of process R&D, β, measures the applicability of knowledge gained from process R&D project for one product to R&D projects for other, related products within the same firm.

that how process R&D investment varies with the number of products depends on the value of β. To this end, I here consider the case that $\beta = 0$ (no scope economies in R&D) and show that, opposite to the situation that $\beta = 1$ the monopolist's incentive for cost-reducing R&D in this case is inversely related to the number of products being offered.[3]

With product specific cost-reducing R&D, the optimal output mix $Q = (q_1, q_2, \ldots, q_n)$ for the monopolist is the solution to

$$\max \pi_m = \sum_{i=1}^{n} \left[\alpha - q_i - \gamma \sum_{j \neq i} q_j \right] q_i - \sum_{i=1}^{n} c(k_i) q_i - \sum_{i=1}^{n} \xi k_i^2 - \theta n F \tag{5}$$

The first order conditions are

$$\alpha - 2q_i - 2\gamma \sum_{j \neq i} q_j = c_i, \quad i = 1, 2, \ldots, n \tag{6}$$

where $c_i \equiv \bar{c} - k_i$. Summing up the first-order conditions over i, we get

$$\sum_{i=1}^{n} q_i = \frac{n\alpha - \sum_{i=1}^{n} c_i}{2(1-\gamma)}$$

Substituting this back into (6) and rearranging terms, we obtain the equilibrium output levels given R&D portfolio $(k_1, k_2, \ldots, k_n)$

$$q_i = q_i(k_1, k_2, \ldots, k_n) = \frac{(1-\gamma)\alpha - [1 + (n-2)\gamma] c_i + \gamma \sum_{j \neq i} c_j}{[2(1-\gamma)][1 + (n-1)\gamma]}$$

If $k_i = k$ for all i, then

$$q_i = \frac{\alpha - \bar{c} + k}{2(1 + (n-1)\gamma)} \tag{7}$$

The equilibrium profits of the monopolists, by noting (6), are

$$\pi_m^* = \sum_{i=1}^{n} \left[q_i + \gamma \sum_{j \neq i} q_j \right] q_i - \sum_{i=1}^{n} \xi k_i^2 - \theta n F$$

The following fact is obvious and used in deriving the R&D solution later.

[3] Lin (2003) compares a multiproduct monopolist' incentive for cost-reducing R&D with that of single-product oligopolist for the general case that $0 \leq \beta \leq 1$.

Fact 1 For any $(k_1,k_2,\ldots,k_n)$, there are

$$\frac{\partial q_i}{\partial k_i}=\frac{1}{2(1-\gamma)}\frac{[1+(n-2)\gamma]}{1+(n-1)\gamma}>0,\quad \text{for all } i$$
$$\frac{\partial q_i}{\partial k_j}=\frac{1}{2(1-\gamma)}\frac{-\gamma}{1+(n-1)\gamma}<0,\quad \text{for all } j\neq i$$

Next, we examine the monopolist's process R&D decisions.

Anticipating the relationship of $q_i(k_1,k_2,\ldots,k_n)$, the monopolist maximizes π_m^* by choosing $(k_1,k_2,\ldots,k_n)$. To see more clearly the first order conditions for k_s, rewrite π_m^* as

$$\pi_m^*=\left(q_s^2+2\gamma\sum_{j\neq s}q_sq_j\right)+\left(\sum_{i\neq s}^{n}q_i^2+\gamma\sum_{i\neq s}q_i\left(\sum_{j\neq i,j\neq s}q_j\right)\right)-\sum_{i=1}^{n}\xi k_i^2-\theta nF$$

Denote the terms in the first and the second parentheses as A and B respectively. Then, noting Fact 1, we have

$$\begin{aligned}\frac{\partial A}{\partial k_s}&=2q_s\frac{\partial q_s}{\partial k_s}+2\gamma\sum_{j\neq s}\left(q_j\frac{\partial q_s}{\partial k_s}+q_s\frac{\partial q_j}{\partial k_s}\right)\\&=2q_s\frac{1}{2(1-\gamma)}\frac{[1+(n-2)\gamma]}{1+(n-1)\gamma}+2\gamma\left(\sum_{j\neq s}q_j\right)\frac{1}{2(1-\gamma)}\frac{[1+(n-2)\gamma]}{1+(n-1)\gamma}\\&\quad+2\gamma q_s\frac{(n-1)}{2(1-\gamma)}\frac{(-\gamma)}{1+(n-1)\gamma}.\end{aligned}$$

We focus on symmetric solution so that $k_s=k$ In this case post-R&D output levels are also the same: $q_s=q$. Thus, at the symmetric solution $\partial A/\partial k_s$ simplifies to

$$\frac{\partial A}{\partial k_s}=q\frac{[1+(n-1)\gamma][1+(n-2)\gamma]-(n-1)\gamma^2}{(1-\gamma)[1+(n-1)\gamma]}$$

where q is given by (7).

The derivative of the terms in the second parenthesis of π_m^* is

$$\frac{\partial B}{\partial k_s}=\sum_{i\neq s}^{n}2q_i\frac{\partial q_i}{\partial k_s}+\gamma\sum_{i\neq s}\left(\frac{\partial q_i}{\partial k_s}\left(\sum_{j\neq i,j\neq s}q_j\right)+q_i\sum_{j\neq i,j\neq s}\frac{\partial q_j}{\partial k_s}\right)$$

At the symmetric R&D solution, there is

$$\frac{\partial B}{\partial k_s} = \frac{2q(n-1)}{2(1-\gamma)} \frac{-\gamma}{1+(n-1)\gamma} + \frac{2q(n-1)(n-2)\gamma}{2(1-\gamma)} \frac{-\gamma}{1+(n-1)\gamma}$$
$$= \frac{-\gamma q(n-1)[1+(n-2)\gamma]}{(1-\gamma)[1+(n-1)\gamma]}.$$

Therefore, the first order condition for the symmetric R&D solution is

$$\frac{\partial \pi_m^*}{\partial k_s} = \frac{\partial A}{\partial k_s} + \frac{\partial B}{\partial k_s} - 2\xi k = 0$$

which further simplifies to

$$q = \frac{\alpha - \bar{c} + k}{2(1+(n-1)\gamma)} = 2\xi k$$

We thus have

$$k_i = k^* = \frac{\alpha - \bar{c}}{4\xi[1+(n-1)\gamma] - 1}$$

for all the products the monopolist offers. Obviously, k^* decreases with n.

Lemma 2 Assume that there are no economies of scope in process R&D (i.e., $\beta = 0$). Then, the monopolist's incentive towards process innovation is decreasing in the number of products being supplied.

5. Concluding remarks

This note has corrected an error in Lemma 1 of Lambertini (2003). That error however does not undermine the main contribution of Lambertini. Among other things, his main insight that monopoly causes distortions in both dimensions of R&D is independent of his claim in Lemma 1. The author's insight that a regulator aiming at correcting distortion in one such dimension should not ignore distortions in the other dimension also makes sense. Of course, in light of the results obtained here (Lemma $1'$ and Lemma 2), the statement on p.572 of Lambertini should be taken to mean that whether subsidization of product innovation would cause reduction of the monopolist's process R&D depends on the degree of economies of scope in process R&D.

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