Product market competition and R&D rivalry

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Abstract

This paper analyzes the impact of a future R&D race on current firm behavior in the product market. It is shown that, in order to “soften” rivals in the future R&D race, firms behave less aggressively in the pre-innovation product market than in the standard duopoly models. As a result, the R&D rates are lower than what firms would choose if standard duopoly outcomes are assumed. R&D cooperation is shown to be capable of restoring the duopoly outcome. © 1998 Elsevier Science S.A.

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1. Introduction

It is well-known in the industrial organization literature that firms’ incentive for R&D depends crucially on their current profits versus the rent innovations will confer. In the literature, firms’ current profits are typically assumed to be exogenous: They are assumed to be either the Cournot profits or the Bertrand profits as in the standard oligopoly models without innovations. This paper makes the point that if current profits are made endogenous, then firms compete less aggressively than in standard oligopoly models in order to reduce the intensity of future R&D rivalry.

The model I consider is a two-stage game whereby two firms compete in the product market in the first stage of the game, and engage in a Lee-Wilde type R&D race in the second stage. First stage actions determine firms’ flow profits during the R&D race, which in turn affect their R&D intensities. It is shown that the equilibrium prices prior to innovation are higher than in the standard duopoly level. The R&D rate is lower than what the firms would choose if the standard duopoly equilibrium outcome is assumed. R&D cooperation can restore the standard duopoly outcome. It does so by reducing the threat R&D rivalry imposes on the current product market. Thus, cooperative R&D may increase welfare in the pre-innovation market, a virtue not yet recognized in the literature.
The first stage actions are interpreted as output or pricing decisions in this model. They can also be viewed as building plants of given size, choosing the number of service outlets, or advertising, etc. The specification of the timing of the game, namely that the firms choose actions in the current product market first and then the intensity of R&D, is designed to capture the idea that the degree of current competition influences incentive for future R&D. The more furiously firms compete currently, the more eager they are to come up with the next generation of technology. Instead of a two-stage game, one can consider alternatively a model where decisions regarding both the product market and R&D are made repeatedly over time. In such a situation, firms basically play the same game at each point of time as long as no innovation has occurred. Because of the stationarity, the analysis of this paper can still go through. However, additional equilibria may arise in such repeated games, such as the collusive outcomes supported by the standard “trigger strategies”. The point of this paper is that firms can “collude” in the product market without launching price wars, but instead by using R&D threats. Finally, if firms interact in an environment where R&D decisions cannot be adjusted very easily but output decisions can, the result of the paper may not hold. This is because that once R&D decisions are set, firms can compete more aggressively without fearing the R&D retaliations.

This paper is related to Reinganum (1983) who showed that an incumbent monopolist is more likely to lose the R&D race to a potential entrant. Cabral (1995) studies how cooperative R&D affects profits in the current product market. Cabral showed that firms may deliberately lower their current prices, thereby increasing their incentive for R&D, in order to turn unstable R&D agreements into stable ones. In a model with a sequence of innovations, Reinganum (1985) examined the feedback from later-stage R&D to an earlier stage race. In her model, a firm’s incentive to research in the current race is reduced, as the winner of the race is more likely to lose the next one. This is similar to the situation in my model where firms’ willingness to compete in the current market is diminished as they foresee that earning a higher profit means a more likely defeat in future R&D race.

2. The model

Two firms play the following two-stage game. In the first stage, they simultaneously and independently take actions, denoted as $a_i$, regarding the current product market. In a Cournot set-up, $a_i$ represents output, whereas in a Bertrand game it is price. In the second stage of the game, the firms engage in an R&D race, the winner of which is awarded a prize $V$ and the loser gets nothing. The R&D specification is drawn from Lee and Wilde (1980). In particular, the probability of discovery by time $t$ for a firm investing at a rate $x_i$ is $1 - e^{-h(x_i)t}$. Firms’ flow profits prior to a discovery are determined by their actions and are denoted by $H_i (a_1, a_2)$. A strategy of firm $i$ in this entire game can then be written as $s_i = (a_i, x_i(\cdot))$, where $x_i(\cdot)$ is a function specifying firm $i$’s R&D rate conditional on firststage actions. Given $(s_1, s_2)$, firm $i$’s payoff structure is standard:

$$W_i(H_i, x_1, x_2) = \int_0^{+\infty} e^{-rt} e^{-[h(x_1)+h(x_2)]} [H_i + Vh(x_i) - x_i] dt = \frac{H_i + Vh(x_i) - x_i}{r + h(x_1) + h(x_2)}.$$
where $r$ is the discount rate and $\Pi_i = \Pi_i(a_1, a_2)$. For simplicity, I assume that $\Pi_i (a_1, a_2) = \Pi_i (a_2, a_1)$ (i.e., the firms are symmetric in the current product market) and that $\Pi_i (a_1, a_2)$ is strictly concave in $a_i$.

### 3. Subgame perfect equilibrium

To analyze the subgame perfect equilibrium of the game, I first study the R&D race.

#### 3.1. The R&D stage

Given $(\Pi_1, \Pi_2)$, firm $i$ maximizes $W_i$ over $x_i$ in the R&D stage. The first order condition of the problem defines the R&D best response function $x_i(x_j, \Pi_i)$. By differentiating the first-order condition, one easily obtains that $\partial x_i / \partial x_i > 0$ and $\partial x_i / \partial \Pi_i < 0$ (see the Appendix). The first property says that the R&D reaction curves are upward sloping. The second property states that a firm’s R&D reaction curve shifts downwards as its preinnovation profit rises. As $\Pi_i$ rises, the net gain from innovation declines, so firm $i$ invests less in R&D. This corresponds to Arrow (1962) which argued that a monopolist has less incentive to innovate than a competitive firm since the former replaces itself when innovating. Tirole (1988) called this the “replacement effect”.

Let $x_i^*(\Pi_1, \Pi_2)$ denote the Nash equilibrium R&D rates, where $\Pi_i = \Pi_i (a_1, a_2)$. The following lemma states how $x_i^*$ depends upon the first-stage decisions.

**Lemma 1.** Assume that the R&D equilibrium $(x_1^*, x_2^*)$ is stable. Then,

\[
(i) \quad \frac{\partial x_i^* (\Pi_1, \Pi_2)}{\partial \Pi_i} < 0, \quad \text{(the own - replacement effect)}
\]

\[
(ii) \quad \frac{\partial x_j^* (\Pi_1, \Pi_2)}{\partial \Pi_i} < 0, \quad \text{(the cross - replacement effect)}
\]

and (iii) as a function of $a$, $x_i^* (\Pi_1 (a, a), \Pi_2 (a, a))$ is strictly decreasing when $a < a^m$ and strictly increasing when $a > a^m$, where $a^m = \arg \max \Pi_i (a, a)$.

Property (i) of the lemma is a direct implication of the “replacement effect”. An increase in $\Pi_i$ shifts firm $i$’s reaction curve downwards, thereby reducing its R&D level. Property (ii) may be called the “cross-replacement effect.” As firm $i$ lowers its R&D expenditure, firm $j$ reacts by cutting its R&D effort as well. (Recall that R&D reaction curves are upward sloping.) The maximizer $a^m$ defined in part (iii) of Lemma 1 corresponds to the cartel solution in the current product market. In a Cournot set-up, $a^m$ equals half the monopoly output, and in a Bertrand model $a^m$ represents the collusive price. According to this part of the lemma, the (symmetric) R&D equilibrium is minimized at $a^m$, given that firms take identical actions in stage 1 of the game. The intuition is simple: Since R&D incentive is inversely related to the current profits, the closer the firms’ action to the cartel solution $a^m$, the lower the R&D rate.
3.2. The pre-innovation product market

In stage 1 of the game, firms optimize over their actions, knowing that the second stage equilibrium is going to be \((x_1^*, x_2^*)\). Given \((a_1, a_2)\), firm \(i\)’s over-all payoffs can be written in the reduced form: 

\[ W_i = W_i(I_i, x_i^*, x_2^*). \]

Differentiating \(W_i\) with respect to \(a_i\) (and noting that the effect of a change in \(x_i^*\) on \(W_i\) can be ignored by the Envelope Theorem) yields

\[
\frac{\partial W_i}{\partial a_i} = \frac{\partial W_i}{\partial I_i} \frac{\partial I_i}{\partial a_i} + \frac{\partial W_i}{\partial x_i^*} \frac{\partial x_i^*}{\partial a_i} + \frac{\partial W_i}{\partial x_j^*} \frac{\partial x_j^*}{\partial a_i} + \frac{\partial W_i}{\partial a_i} \left[ \frac{\partial x_i^*}{\partial a_i} + \frac{\partial x_j^*}{\partial a_i} \right], \quad i \neq j, i, j = 1, 2
\]

The first term is the direct effect of firm \(i\)’s action on its payoff. A change in \(a_i\) also affects the R&D game. In particular, a firm’s first-stage action alters rival firm’s R&D level \(x_i^*\), which will in turn impact on \(W_i\). This indirect effect is presented by the second term of the above expression. Recognizing this indirect effect, firm \(i\) optimizes by setting \(\partial W_i/\partial a_i = 0\), which is equivalent to

\[
\frac{\partial I_i}{\partial a_i} = - \frac{\partial W_i}{\partial I_i} \frac{\partial x_i^*}{\partial a_i} - \frac{\partial W_i}{\partial x_j^*} \frac{\partial x_j^*}{\partial a_i} - \frac{\partial W_i}{\partial a_i} \left[ \frac{\partial x_i^*}{\partial a_i} + \frac{\partial x_j^*}{\partial a_i} \right], \quad i \neq j, i, j = 1, 2. \tag{1}
\]

Eq. (1) implicitly defines firm \(i\)’s first-stage best response function, which we denote as \(R(a_i)\). The denominator is positive because \(W_i\) is positively related to \(I_i\) and negatively related to \(x_i\), and because \(\partial x_i^*/\partial I_i < 0\) (the cross-replacement effect). The numerator has the same sign as \(\partial I_i/\partial a_i\), since \(\partial x_i^*/\partial I_i > 0\) (the own-replacement effect). Thus, at \(R(a_i)\), \(\partial I_i/\partial a_i\) and \(\partial I_i/\partial a_i\) have opposite signs.

In a Cournot model, \(a_i\) will represent output, \(q_i\). In this case, \(\partial I_i/\partial q_i < 0\) if \(q_i > 0\), and \(\partial I_i/\partial q_i = 0\) if \(q_i = 0\). (An increase in a firm’s output always reduces rival’s profits unless \(q_j = 0\).) The property of the best response function in this model can be easily learned from expression (1). Specifically, if \(q_j > 0\), then \(\partial I_i(q_1, q_2)/\partial q_i\) is positive at \(q_i = R(q_i)\). So, \(R(q_j)\) is smaller than its counterpart in the standard Cournot model. If \(q_j = 0\), then \(\partial I_i(q_1, q_2)/\partial q_i = 0\) at \(q_i = R(q_i)\). In this case, a firm’s best response equals the monopoly output \(Q^m\), as in the standard Cournot case. Therefore, a firm’s reaction curve in this model lies below the reaction curve in the standard Cournot set-up.

If firms produce differentiated products and engage in price competition in the first stage of the game, then \(a_i\) represents price, \(p_i\). It follows that \(\partial I_i/\partial p_i > 0\) for any \(p_i\) and \(p_j\); a price increase by a firm benefits its rival. Then, expression (1) implies \(\partial I_i/\partial p_i < 0\) at firm \(i\)’s best response to \(p_j\), \(R(p_j)\). That is, firms overprice in stage 1 of the game. Given the rival’s price, firm \(i\) sets its price at a level where its profit function \(I_i\) is declining. Thus, the first-stage reaction curves lie above those in Bertrand models without R&D. The following proposition then obtains.

**Proposition.** Under Cournot or Bertrand competition, the equilibrium price in the model is strictly above the standard duopoly level.

\(^{1}\)We have so far ignored possible corner solutions. If for certain output level of firm \(j\), firm \(i\)’s best response is zero, the statement automatically holds.
4. R&D cooperation

This section makes the point that R&D cooperation can restore the standard duopoly outcome in the product market.

Under R&D cooperation, the industry solves the following problem in the R&D stage for any pre-innovation actions \((a_1, a_2)\),

\[
\text{Maximize } W_1 + W_2 = \frac{II_1 + II_2 + V[h(x_1) + h(x_2)] - x_1 - x_2}{r + h(x_1) + h(x_2)}. \quad (x_1, x_2)
\]

The first order conditions can be written as \(W_1 + W_2 = V - 1/h'(x_1)\) and \(W_1 + W_2 = V - 1/h'(x_2)\). Thus, the cooperative solution entails equal R&D investment by both firms. Let \(\hat{x}(\Pi)\) denote the solution, where \(\Pi \equiv II_1 (a_1, a_2) + II_2 (a_1, a_2)\) represents the industry profits.

As in Section 3, firms still behave non-cooperatively in stage one of the game. Anticipating that \(\hat{x}\) will be the (cooperative) R&D level, firm \(i\) solves

\[
\text{Maximize } \hat{W}_i(a_1, a_2) = \frac{II_i + Vh(\hat{x}) - \hat{x}}{r + 2h(\hat{x})}. \quad a_i
\]

The first-order condition is

\[
\frac{\partial II_i}{\partial a_i} = [2h'\hat{W}_i - (Vh' - 1)] \frac{\partial \hat{x}}{\partial a_i}, \quad \text{where} \quad \frac{\partial \hat{x}}{\partial a_i} = \frac{\partial \hat{x}}{\partial II} \left( \frac{\partial II_i}{\partial a_i} + \frac{\partial II}{\partial a_i} \right).
\]

which can be rewritten, using the first-order conditions for R&D, as

\[
\frac{\partial II_i}{\partial a_i} = (\hat{W}_i - \hat{W}_j)h' \frac{\partial \hat{x}}{\partial a_i}.
\]

Thus, the symmetric equilibrium action under R&D cooperation, denoted as \(\hat{a}\), is determined by the equation

\[
\frac{\partial II(\hat{a}, \hat{a})}{\partial a_i} = [\hat{W}_i(\hat{a}, \hat{a}) - \hat{W}_j(\hat{a}, \hat{a})]h'(\hat{x}) \frac{\partial \hat{x}}{\partial a_i}.
\]

Since

\[
\hat{W}_i(\hat{a}, \hat{a}) = \hat{W}_j(\hat{a}, \hat{a}), \quad \frac{\partial II(\hat{a}, \hat{a})}{\partial a_i} = 0.
\]

Therefore, \(\hat{a}\) coincides with the traditional duopoly equilibrium action.

5. Conclusions

This paper studies how subsequent R&D influences current firm behavior in the product market. In a two-stage game, firms compete in the pre-innovation product market first and then engage in a patent race. In order to “soften” rivals’ incentive for R&D, firms adopt the “puppy dog” strategy
(Fudenberg and Tirole (1984): they behave less aggressively in the product market than in standard duopoly models. Consequently, equilibrium price is higher than in standard duopoly models, and the pace of innovation is slower than if the standard duopoly equilibrium outcome is imposed. If firms can coordinate their R&D decisions, then the standard duopoly equilibrium obtains, as the R&D threat that gives rise to “collusive” conduct is eliminated.

The welfare analysis of the results of the paper is ambiguous. Although a price increase in the current product market raises deadweight loss prior to a discovery, a lower R&D investment level may be welfare improving, since a patent race in general generates excessive R&D effort. By the same token, R&D cooperation, which reduces price to the duopoly level, may or may not increase social welfare. To learn more about the welfare property of the equilibrium and of cooperative R&D in this model, one needs specific information on the demand condition, the nature of the R&D technology, the difference between the social value and the private value of the patent, etc.

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Appendix 1

**Proof of Lemma 1:** A firm’s R&D best response is defined by the first-order condition:

\[ G(x_i, x_j, \Pi_i) = [Vh'(x_i) - 1][r + h(x_j)] - h(x_i) + x_i h'(x_i) - \Pi_i h'(x_i) = 0. \]

Let \( G_i \) denote the partial derivatives of \( G \) with respect to its \( k \)th argument, \( k = 1, 2 \). Then, \( G_i = \left[ (rV - \Pi_i + Vh(x_i) + x_i)h''(x_i), \right. \) and \( G_2(x_i, x_j, \Pi_i) = [Vh'(x_i) - 1]h'(x_i). \) It is easy to see that \( Vh'(x_i) - 1 > 0 \). This follows from the above first-order condition and the fact that \( h(x) > xh'(x) \) (assuming \( h \) is concave and \( h(0) = 0 \)).

The stability requirement of R&D equilibrium amounts to assuming: For any \((a_1, a_2), G_1(x_1^*, x_2^*, \Pi_1) > G_2(x_1^*, x_2^*, \Pi_2) > G_3(x_1^*, x_2^*, \Pi_3) \), where \( x_1^* = x_1^*(\Pi_1, \Pi_2) \) and \( \Pi_1 = \Pi_1(a_1, a_2), i = 1, 2 \). When \( \Pi_1 = \Pi_2 \), this inequality is equivalent to \( G_1 + G_2 < 0 \), which is the stability condition common assumed in the literature (Tirole, 1988).

The R&D equilibrium is defined by \( G(x_1^*, x_2^*, \Pi_i) = 0 \) and \( G(x_1^*, x_2^*, \Pi_i) = 0 \).

Differentiating both equations with respect to \( \Pi_i \) and solving for \( \frac{\partial x_1^*}{\partial \Pi_i} \) and \( \frac{\partial x_2^*}{\partial \Pi_i} \), yields

\[ \frac{\partial x_1^*}{\partial \Pi_i} = h'(x_1^*)G_1(x_1^*, \cdot) / \Delta \] and \[ \frac{\partial x_2^*}{\partial \Pi_i} = -h'(x_2^*)G_2(x_2^*, \cdot) / \Delta, \]

\(^5\)At \((x_1, x_2), \) the slope of firm 1’s reaction curve is \(-G_1(x_1, x_2, \Pi_1)/G_2(x_1, x_2, \Pi_1)\), and that of firm 2’s is \(-G_2(x_2, x_1, \Pi_2)/G_1(x_2, x_1, \Pi_2)\).
where $\Delta = G_1(x^*, \cdot)G_1(x^*, \cdot) - G_2(x^*, \cdot)G_2(x^*, \cdot)$. Since $\Delta > 0$ by the stability condition, we have $(dx_i^*/d\Pi_i) < 0$ and $(dx_j^*/d\Pi_j) < 0$. This proves (i) and (ii).

At $x_i^* = x_j^*(\Pi_1(a, a), \Pi_2(a, a))$, there is $G(x_i^*, x_j^*, \Pi_i) = 0$. Differentiating both sides of the equation with respect to $a$ yields

$$\frac{dx_i^*}{da} = \frac{h'(x_i^*)}{G_1 + G_2} \frac{d\Pi_i(a, a)}{da}.$$

Since $G_1 + G_2 < 0$, $dx_i^*/da$ and $d\Pi_i/da$ have opposite signs. This proves part (iii) of the lemma. Q.E.D.

References


