

The Retail Space-Exchange Problem with Pricing and Space Allocation Decisions

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We consider retail space-exchange problems where two retailers exchange shelf space to increase accessibility to more of their consumers in more locations without opening new stores. Using the Hotelling model, we find two retailers' optimal prices, given their host and guest space in two stores under the space-exchange strategy. Next, using the optimal space-dependent prices, we analyze a non-cooperative game, where each retailer makes a space allocation decision for the retailer's own store. We show that the two retailers will implement such a strategy in the game, if and only if their stores are large enough to serve more than one-half of their consumers. Nash equilibrium for the game exists, and its value depends on consumers' utilities and trip costs as well as the total available space in each retailer's store. Moreover, as a result of the space-exchange strategy, each retailer's prices in two stores are both higher than the retailer's price before the space exchange, but they may or may not be identical.

Key words: retail space-exchange; price; space allocation; Hotelling model; Nash equilibrium

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1. Introduction

Can retailers selling different products implement partnership strategies that involve exchanging shelf space to improve their operating performance? Consider the following problem (and its solution) that was experienced by the British supermarket chain of food-related products known as *Waitrose* (The Waitrose Press Center 2009): Even though its sales were increasing, many of Waitrose's potential customers were having difficulty accessing its stores because no new stores were being opened. To increase accessibility to more of its customers in more locations without opening new stores, Waitrose established new channels and implemented a new business model. This was achieved by initiating strategic relationships with the British retailer of healthcare products known as *Boots*. Waitrose and Boots now stock "selective product ranges" in each others' stores; more specifically, Waitrose's food products are sold in Boots's stores, while the latter retailer's healthcare products are displayed for sale in Waitrose stores. The cooperation between Waitrose and Boots can be regarded as an

implementation of the retail space-exchange strategy; see Stych (2009) for a magazine article describing this partnership.

Waitrose and Boots have successfully implemented the space-exchange strategy, as indicated in a report by The Waitrose Press Center (2009). As another successful example of this novel strategy, Canada's favorite doughnut store known as *Tim Hortons* has been working with the US-based Cold Stone Creamery (a chain of ice cream stores) to implement the space-exchange strategy and operate their "co-branded" stores. This practice involves 100 stores in the United States and six in Canada. For more information, see Draper (2009) for an article describing the partnership between Tim Hortons and Cold Stone Creamery.

With the novel retailing practice described above, when two retailers (say, 1 and 2) implement their space-exchange strategy, a retailer's consumers can buy in either the retailer's own store or the other retailer's store, which means that such a strategy can result in an increased store choice for consumers. It is reasonable to expect that each consumer would buy in a store that is closer to his or her residence location, thereby

incurring lower travel costs and increasing the willingness to buy at a higher retail price. Specifically, when two retailers do not exchange their space, consumers who intend to buy a retailer's product will have to visit the retailer's own store; however, after the space exchange, some consumers may decide to shop at the other retailer's store (because it is closer to those consumers). This implies that the space-exchange strategy will reduce the travelling costs of some consumers who buy a retailer's product at the other retailer's store. As each consumer does not need to incur a high travelling cost, he or she should be willing to buy even if the retail price is increased. Thus, both retailers 1 and 2 may respond by raising their prices to increase their profit margins. This can be regarded as the most important benefit derived from the space-exchange strategy. We find that each retailer's pricing and space allocation decisions are important to the success of the space-exchange strategy, which are the focused research questions in our article.

One may question why the two retailers do not sell both products 1 and 2 by themselves in their own stores, but instead, exchange shelf space for the sale of these products. We present three reasons for this: First, when the two retailers sell identical products at their sites, they have to compete for consumers, which may result in the reduction of the two retailers' profits. Second, the two retailers have more information about their own products and are thus specialized in their product sales. If each retailer sells his own product and also the other retailer's product, then the retailer has to allocate his efforts for the sale of the product that is unfamiliar to the retailer; this may reduce the retailer's operational profitability. Third, as the retailers should have already served their markets before the space exchange, their established reputations may affect consumers' purchasing decisions. Hence, to reduce the operational risk, each retailer should optimally allocate his space to the other retailer and thus take advantage of the other retailer's reputation to efficiently provide more choices to consumers.

As the above discussion indicates, the space-exchange strategy should generate benefits to both retailers and their consumers; and, in practice, some retailers (e.g., Waitrose and Boots, Tim Hortons and Cold Stone Creamery) have already successfully implemented this strategy. Despite the apparent importance of this strategy, our literature search did not reveal any research articles dealing with space-exchange problems. There are a number of space-related publications, which do not consider the space exchange-related issues. In one publication that is closely related to our article, Jerath and Zhang (2010) consider a store-within-a-store arrangement, in which a retailer rents out her retail space to two manufacturers who then have complete autonomy over retail

decisions such as pricing and in-store service. The authors develop an analytical model to investigate the retailer's trade-off between channel efficiency and interbrand competition. They show that the retailer cannot credibly commit to the retail prices and service levels that two manufacturers can achieve in an integrated channel, and she should thus allow the manufacturers to set up stores within her store. In another related publication, Martínez-de-Albéniz and Roels (2011) analyze a shelf-space competition problem where a single retailer optimizes her shelf space allocation among multiple suppliers' different products, based on their sales level and profit margins. The authors examine the equilibrium situation in the supply chain, and find that, in general, the retailer's and the suppliers' incentives are misaligned, resulting in suboptimal retail prices and shelf space allocations. Other recent representative space-related publications include Baron et al. (2011), Campo et al. (2000), Kurtuluş and Toktay (2011), and Wang and Gerchak (2002).

In this article, we use the Hotelling model (Hotelling 1929) in section 2 to analyze the space-exchange problem where retailers 1 and 2 are located at two end points of a linear city. For detailed discussions on the Hotelling model and its extensions, see, e.g., Martin (1993) and Tirole (1988). The Hotelling model has been widely used to analyze marketing- and operations management-related problems. The recent representative publications, where the Hotelling models are considered, include, e.g., Dasci and Laporte (2005), Ghosh and Balachander (2007), Granot et al. (2010), and Sajeesh and Raju (2010).

In our space-exchange problem, retailer i ($i = 1, 2$) sells product i to his consumers who are uniformly distributed between the two retailers. As the success of the space-exchange strategy naturally depends on whether or not each retailer benefits from this strategy, we begin our analysis by finding each retailer's optimal pricing decision and maximum profit when they do not exchange shelf space, which are later compared with two retailers' profits under the space-exchange strategy. Next, when the two retailers decide to exchange shelf space, we first temporarily assume that, in each store, two retailers have sufficiently large space to serve all of their consumers, and calculate their corresponding optimal prices with no space (capacity) constraint.

We then find in section 3 each retailer's optimal prices under the space constraint, i.e., the retailer's host space in his own store and his guest space allocated by the other retailer are arbitrarily given. Using two retailers' optimal space-dependent prices, we next analyze in section 4 a non-cooperative game where each retailer maximizes his total profit in two stores to determine optimal space allocation decisions for his own store and find the corresponding optimal

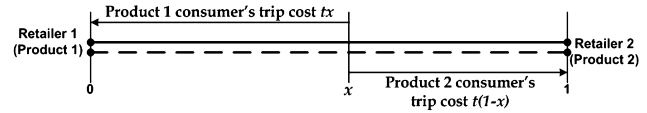
prices for his product in two stores. We perform our best-response analysis for two retailers, and find that Nash equilibrium for the game may or may not uniquely exist, which depends on consumers' consumption utilities, trip cost, and the total space in each store. We show that, adopting the Nash equilibrium, each retailer can achieve a higher profit than before the space exchange. In section 5, we discuss possible changes of our major results in the presence of a "common" consumer who buys both products 1 and 2, or those when a retailer's fixed cost of opening and staffing a new store is considered.

2. Preliminaries

As indicated by the practice of Waitrose and Boots, and also by that of Tim Hortons and Cold Stone Creamery, the retail space-exchange strategy applies only when the cooperating retailers' products are neither substitutable nor complementary, e.g., Waitrose's food vs. Boots's healthcare products; and, Tim Hortons's doughnuts vs. Cold Stone Creamery's ice cream. Thus, we can reasonably assume that the products in categories $i = 1, 2$ sold by retailer $i = 1, 2$, are neither substitutable nor complementary.

The total shelf space that is owned by retailer i is denoted by $S_i > 0$ for $i = 1, 2$. To implement the space-exchange strategy, retailer i —who sells product i —decides to allocate the retail space $S_{ij} \in [0, S_i]$ to retailer j ($j = 1, 2$ and $j \neq i$) who can then sell product j using the space S_{ij} at the site of retailer i as the "guest retailer." As a result of the space exchange, retailer i sells product i in the remaining space $S_{ii} \equiv S_i - S_{ij}$ at his own store as the "host retailer." As discussed in section 1, when two retailers exchange shelf space, their customers may incur lower travel costs, and the two retailers may thus increase their retail prices without losing customers. This may be regarded as an important reason why retailers (e.g., Waitrose and Boots, Tim Hortons and Cold Stone Creamery) exchange shelf space. Accordingly, we consider the Hotelling model (Dasci and Laporte 2005, Ghosh and Balachander 2007, Granot et al. 2010, Hotelling 1929, Sajeesh and Raju 2010) to analyze our space-exchange problem, assuming that two retailers are located at the end points of a "linear city" of length 1, and all consumers are uniformly distributed along the city.

Figure 1 The Trip Cost of Product 1 Consumers (Who Are Served by Retailer 1) and that of Product 2 Consumers (Who Are Served by Retailer 2)



Note that the solid and dashed lines between the two retailers represent the uniform distribution of product 1 consumers and that of product 2 consumers, respectively.

before and after the exchange of shelf space. We next begin by computing two retailers' optimal prices and corresponding maximum profits when they do not exchange shelf space but only operate in their own stores.

2.1. Optimal Pricing Decision with No Space Exchange

When retailers 1 and 2 do not exchange shelf space, they sell products 1 and 2 at the retail prices p_1 and p_2 , respectively. Total number of consumers for product i is B_i , for $i = 1, 2$. In this article, we assume that there is no "common" consumer who intends to buy both products 1 and 2; that is, B_1 and B_2 are disjoint. In section 5.1, we will discuss the impacts of relaxing such an assumption on our major results. As in the Hotelling model, each consumer incurs the transportation cost t per unit of trip length, which includes the consumer's value of time. Let $x \in [0, 1]$ denote a point in the linear city. Assuming that the locations of retailers 1 and 2 are $x = 0$ and $x = 1$, respectively, we can calculate the trip cost of the product 1 consumer (who is served by retailer 1) at the point $x \in [0, 1]$ as tx , and also compute that of the product 2 consumer (who is served by retailer 2) at the point $x \in [0, 1]$ as $t(1 - x)$; see Figure 1. In addition, each product i ($i = 1, 2$) consumer is assumed to draw a gross utility $\bar{u}_i$ from buying a unit of product i .

Using the above, we find that the product 1 consumer at the point $x \in [0, 1]$ obtains the utility $\bar{u}_1$, but incurs the purchase cost p_1 and the trip cost tx , and the product 2 consumer at the point $x \in [0, 1]$ gets the utility $\bar{u}_2$, but incurs the purchase cost p_2 and the trip cost $t(1 - x)$. It thus follows that the net utility function of the consumer at location x is calculated as,

$$u_x = \begin{cases} u_{x1} \equiv \bar{u}_1 - p_1 - tx, & \text{if product 1 bought in retailer 1's store,} \\ u_{x2} \equiv \bar{u}_2 - p_2 - t(1 - x), & \text{if product 2 bought in retailer 2's store.} \end{cases} \quad (1)$$

As the two retailers are willing to exchange shelf space when they can enjoy more profits from the strategy, we need to compare the two retailers' profits

A product 1 consumer should be willing to buy from retailer 1 if $u_{x1} \geq 0$, or, $x \leq x_1 \equiv (\bar{u}_1 - p_1)/t$. This means that only the product 1 consumers who

are located between 0 (retailer 1's location) and x_1 should decide to buy. Naturally, retailer 1 should set his retail price p_1 such that $0 \leq x_1 \leq 1$, or $\bar{u}_1 - t \leq p_1 \leq \bar{u}_1$. Similarly, retailer 2 should determine her price p_2 such that $\bar{u}_2 - t \leq p_2 \leq \bar{u}_2$.

Then, we can calculate the demand faced by retailer i ($i = 1, 2$) as $D_i = B_i(\bar{u}_i - p_i)/t$. However, each retailer may or may not satisfy his demand, because he only has the space S_i to stock product i . Assuming that each retailer can display one unit of his product on a unit of the retail space, we find that retailer i can realize the sales $\min[B_i(\bar{u}_i - p_i)/t, S_i]$ and achieve the profit as,

$$\begin{aligned}\pi_i &= (p_i - c_i) \min[B_i(\bar{u}_i - p_i)/t, S_i] \\ &= B_i(p_i - c_i) \min[(\bar{u}_i - p_i)/t, S_i/B_i],\end{aligned}\quad (2)$$

where c_i denotes retailer i 's unit acquisition cost. To determine the optimal price p_i^* for retailer i , we must solve the constrained maximization problem, $\max_{\bar{u}_i - t \leq p_i \leq \bar{u}_i} \pi_i$.

LEMMA 1. *When two retailers do not exchange shelf space, the optimal prices and maximum profits for retailer i ($i = 1, 2$) can be found as follows:*

Conditions		Optimal price	Maximum profit
$S_i \geq B_i$	$c_i + 2t \geq \bar{u}_i$	$(\bar{u}_i + c_i)/2$	0
	$c_i + 2t \leq \bar{u}_i$	$\bar{u}_i - t$	$B_i(\bar{u}_i - t - c_i)$
$S_i < B_i$	$c_i + 2tS_i/B_i \geq \bar{u}_i$	$(\bar{u}_i + c_i)/2$	$B_i(\bar{u}_i - c_i)^2/(4t)$
	$c_i + 2tS_i/B_i \leq \bar{u}_i$	$\bar{u}_i - tS_i/B_i$	$(\bar{u}_i - tS_i/B_i - c_i)S_i$

PROOF. For a proof of this lemma and the proofs of all subsequent lemmas, see Appendix S1. $\square$

2.2. Optimal Retail Prices under the Space-Exchange Strategy with Sufficiently Large Host and Guest Spaces

We consider the two retailers' optimal pricing decisions when they decide to exchange shelf space. Now, we temporarily assume that each retailer's host space and guest space are large enough to serve all of the retailer's consumers and, under this assumption, we compute the retailer's optimal prices. After the space exchange, each consumer can buy either in retailer 1's store or retailer 2's store, which depends on which

uct i from retailer i at either his host space [in retailer i 's store] or his guest space [in retailer j 's ($j = 1, 2$ and $j \neq i$) store]. We compute the consumer's utilities drawn from purchasing from two stores as, where $\hat{u}_{xij}$ denotes the product i consumer's net utility drawn from buying at retailer j 's store, and p_{ij} represents the retail price of product i in retailer j 's store.

Similar to section 1, we can compute the demands faced by retailer i in two stores, as given in the following remark. For a detailed discussion, see Appendix S3.

REMARK 1. We find the demands for retailer i 's product ($i = 1, 2$) as follows:

1. If $p_{i1} + p_{i2} \leq 2\bar{u}_i - t$, then the demands faced by retailer i in retailer 1's and retailer 2's stores are computed as $D_{i1} = B_i(p_{i2} - p_{i1} + t)/(2t)$ and $D_{i2} = B_i(p_{i1} - p_{i2} + t)/(2t)$, respectively. Note that $D_{i1} + D_{i2} = B_i$.
2. If $p_{i1} + p_{i2} > 2\bar{u}_i - t$, then the demands faced by retailer i are computed as $D_{i1} = B_i(\bar{u}_i - p_{i1})/t$ and $D_{i2} = B_i(\bar{u}_i - p_{i2})/t$. The total demand for product i is thus $D_{i1} + D_{i2} = B_i(2\bar{u}_i - p_{i1} - p_{i2})/t \leq B_i$.

From the above, we find that all product i consumers will buy when $p_{i1} + p_{i2} \leq 2\bar{u}_i - t$, whereas some consumer(s) may not buy when $p_{i1} + p_{i2} \geq 2\bar{u}_i - t$. Note that, if $p_{i1} + p_{i2} = 2\bar{u}_i - t$, then the demands for the above two cases are the same.

The above remark indicates that retailer i can set suitable retail prices p_{i1} and p_{i2} to affect consumers' purchasing decisions. That is, if the retailer does not have sufficient space in two retailers' stores, then he may determine his retail prices under the condition that $p_{i1} + p_{i2} \geq 2\bar{u}_i - t$. Otherwise, if the retailer's shelf space in the two stores is large enough to satisfy B_i product i consumers, then the retailer may need to consider the condition that $p_{i1} + p_{i2} \leq 2\bar{u}_i - t$ to make his pricing decisions.

LEMMA 2. *When retailer i ($i = 1, 2$) has sufficiently large host and guest shelf space, we find his optimal prices (p'_{i1}, p'_{i2}) and resulting demands (D_{i1}, D_{i2}) in two stores, and compute his maximum profit, as given in Table 1.*

As the above lemma indicates, retailer i may need to determine his prices in two stores to serve some,

$$\hat{u}_{xi} = \begin{cases} \hat{u}_{xi1} \equiv \bar{u}_i - p_{i1} - tx, & \text{if product } i \text{ bought in retailer 1's store,} \\ \hat{u}_{xi2} \equiv \bar{u}_i - p_{i2} - t(1-x), & \text{if product } i \text{ bought in retailer 2's store,} \end{cases} \quad (3)$$

store the consumer can draw a higher net utility. Consider the product i ($i = 1, 2$) consumer who resides at the point $x \in [0, 1]$ and decides to buy a unit of prod-

rather than all, of B_i product i consumers, if the product i consumer residing at the location of retailer j 's store ($j = 1, 2, j \neq i$) cannot enjoy a positive net utility

Table 1 Retailer i 's Optimal Prices p'_{i1} and p'_{i2} , and the Resulting Demands D_{i1} and D_{i2} in Retailer 1's and Retailer 2's Stores, Respectively; and the Retailer's Maximum Profit Generated in the Two Stores

Condition	Optimal prices	Demands	Maximum profit
$\bar{u}_i > c_i + t$	$p'_{i1} = p'_{i2} = \bar{u}_i - \frac{t}{2}$	$D_{i1} = D_{i2} = \frac{B_i}{2}$	$B_i(\bar{u}_i - c_i - \frac{t}{2})$
$\bar{u}_i \leq c_i + t$	$p'_{i1} = p'_{i2} = \frac{\bar{u}_i + c_i}{2}$	$D_{i1} = D_{i2} = B_i \frac{\bar{u}_i - c_i}{2t}$	$B_i \frac{(\bar{u}_i - c_i)^2}{2t}$

from buying product i in retailer i 's store, i.e., $\bar{u}_i \leq c_i + t$.

3. Optimal Prices Given the Space Allocation Decisions

In this section, we consider two retailers' optimal pricing decisions given the space-allocation decisions in two stores. (The results here differ from those in section 2 where we determine the two retailers' optimal prices assuming that they have sufficient host and guest space.) Subsequently, using each retailer's optimal space-dependent prices in two stores, we find the optimal allocation of the total space S_i ($i = 1, 2$) between the two retailers.

Next, we determine retailer i 's ($i = 1, 2$) optimal pricing decisions (p_{i1} and p_{i2}), given the host space S_{ii} in his own store and the guest space $S_{ji} = S_j - S_{jj}$ in retailer j 's ($j = 1, 2, j \neq i$) store. As one unit of product i is carried per unit of shelf space, the total number of product i available for sale in two stores can be calculated as $T_i \equiv S_{1i} + S_{2i}$. Note that each retailer's maximum available products in each store can be regarded as the "capacity" for the retailer, who should thus make his or her optimal pricing decisions under the capacity constraint.

As Lemma 2 indicates, retailer i 's optimal pricing decisions with no capacity constraint depend on the comparison between $\bar{u}_i$ and $c_i + t$. Accordingly, we consider the two cases, (i) $\bar{u}_i > c_i + t$, and (ii) $\bar{u}_i \leq c_i + t$; and for each case, we find retailer i 's optimal prices under the capacity constraints (i.e., at most S_{ii} and S_{ji} units of product i are available for sale in retailer i 's own store and in retailer j 's store).

3.1. Optimal Prices When $\bar{u}_i > c_i + t$

If $\bar{u}_i > c_i + t$, then we learn from Lemma 2 that retailer i should make his pricing decisions to serve all B_i product i consumers, which requires this retailer to have a sufficiently large space to stock B_i units of product i . In addition, as retailer i 's profit is maximized when $S_{ii} = S_{ji} = B_i/2$, his desired host space and guest space should be both equal to $B_i/2$. However, the space allocated to retailer i in each store may be different from $B_i/2$, and the total space for retailer i in two stores may or may not be large enough to serve

all of B_i consumers. More specifically, if the total space for retailer i is given such that $S_{ii} + S_{ji} \geq B_i$, then the total demand B_i can be satisfied; but, if $S_{ii} + S_{ji} < B_i$, then only a part of the demand will be fulfilled.

LEMMA 3. Suppose that retailer i 's ($i = 1, 2$) host space and guest space are given as $S_{ii} \in [0, S_i]$ and $S_{ji} \in [0, S_j]$, for $j = 1, 2$, and $j \neq i$. If $\bar{u}_i > c_i + t$, then retailer i 's optimal prices in two stores are found as follows:

1. If retailer i 's total space $S_{ii} + S_{ji}$ in two stores is large enough to serve all of B_i product i consumers, i.e., $S_{ii} + S_{ji} \geq B_i$, then the retailer's optimal prices in his own store and retailer j 's store—denoted by p_{ii}^* and p_{ij}^* , respectively—are determined as given in Table 2.
2. If $S_{ii} + S_{ji} < B_i$, then the retailer's optimal pricing decisions are obtained as $p_{ii}^* = \max[(\bar{u}_i + c_i)/2, \bar{u}_i - tS_{ii}/B_i]$ and $p_{ij}^* = \max[(\bar{u}_i + c_i)/2, \bar{u}_i - tS_{ji}/B_i]$.

Next, we compare retailer i 's optimal price with no space exchange—as given in Lemma 1—and the optimal price under the space-exchange strategy—as given in Lemma 1 to examine the impact of the strategy on the retailer's pricing decision.

LEMMA 4. When $\bar{u}_i > c_i + t$ ($i = 1, 2$) and retailer i 's host space and guest space are given as $S_{ii} \in [0, S_i]$ and $S_{ji} \in [0, S_j]$ ($j = 1, 2, j \neq i$), respectively, the retailer's optimal price p_{ii}^* in his own store is greater than his optimal price p_i^* that is obtained when there is no space exchange, i.e., $p_{ii}^* > p_i^*$. However, retailer i 's optimal price p_{ij}^* in retailer j 's store may or may not be greater than p_i^* . Specifically, if the total space S_i in retailer i 's own store is larger than or equal to $B_i(\bar{u}_i - c_i)/(2t)$, i.e., $S_i \geq B_i(\bar{u}_i - c_i)/(2t)$, then p_{ij}^* is always greater than p_i^* . Otherwise, then p_{ij}^* may not be greater than p_i^* .

This lemma says that, if retailer i has a sufficiently large space in his own store, i.e., $S_i \geq B_i(\bar{u}_i - c_i)/(2t)$, then his prices in two stores under the space-exchange strategy should be higher than the price when two retailers do not exchange shelf space. More-

Table 2 Retailer i 's Optimal Price p_{ii}^* and p_{ij}^* for Product i in His Host Space $S_{ii} \in [0, S_i]$ and His Guest Space $S_{ji} \in [0, S_j]$, Respectively, When $\bar{u}_i > c_i + t$ and $S_{ii} + S_{ji} \geq B_i$

Conditions	p_{ii}^*	p_{ij}^*
$S_{ii} \geq B_i/2$	$\bar{u}_i - \frac{t}{2}$	$\frac{\bar{u}_i - t}{2}$
$S_{ji} \geq B_i/2$		
$S_{ii} < B_i/2$	$\bar{u}_i - t \frac{S_{ii}}{B_i}$	$\max[\frac{\bar{u}_i + c_i}{2}, \bar{u}_i - t(1 - \frac{S_{ji}}{B_i})]$
$S_{ji} > B_i/2$		
$S_{ii} > B_i/2$	$\max[\frac{\bar{u}_i + c_i}{2}, \bar{u}_i - t(1 - \frac{S_{ji}}{B_i})]$	$\bar{u}_i - t \frac{S_{ji}}{B_i}$
$S_{ji} < B_i/2$		

over, we note that $B_i(\bar{u}_i - c_i)/(2t) > B_i/2$, because $\bar{u}_i > c_i + t$. This means that the prices should rise as a result of the space-exchange strategy, if the total available space S_i in retailer i 's store is large enough to satisfy more than a half of product i consumers (including some consumers who are closer to retailer j 's store). This interesting result may be justified as follows: After the space exchange, those consumers closer to retailer j 's store could visit retailer j 's store to buy product i . That is, retailer i may serve fewer consumers in his own store, and may thus raise his retail price to increase his profit.

REMARK 2. Lemma 3 was used for comparing retailer i 's ($i=1,2$) prices under different conditions. We can use the same lemma to calculate the retailer's maximum profit as given in Table 3.

In section 2.1, we found retailer i 's optimal pricing decision and computed the corresponding maximum profit when the two retailers do not exchange shelf space. We now compare retailer i 's maximum profit (i) with no space exchange and (ii) with space exchange strategy to examine whether or not two retailers can benefit from the strategy.

COROLLARY 1. If the two retailers implement the space-exchange strategy when $\bar{u}_i > c_i + t$, then we find that π_i^1 ($i = 1, \dots, 8, i \neq 6$) in Table 3 is greater than retailer i 's profit when two retailers do not exchange shelf space. However, π_i^6 in Table 3 may be smaller than the retailer's profit with no space exchange.

PROOF. For a proof of this corollary, see Appendix S4. $\square$

As the above corollary indicates, retailer i ($i = 1,2$) may be worse off under the space-exchange strategy, if he cannot use his host space and guest space to serve all of B_i product i consumers (i.e., $S_{ii} + S_{ji} < B_i$), and both the host space and the guest space are smaller than a threshold value [i.e., $S_{ii}, S_{ji} \leq B_i(\bar{u}_i - c_i)/(2t)$]. This means that, to cooperate for such a strategy, retailer i should retain a sufficiently large

host space and retailer j ($j = 1,2, j \neq i$) must also allocate a sufficiently large guest space to retailer i . However, even though π_i^6 in Table 3 may be smaller than the profit with no space exchange, retailer i should still be better off from implementing the space-exchange strategy because he can choose to allocate S_{ii} units to himself. For example, the retailer can increase S_{11} to a level such that his profit is π_i^7 , which is higher than the retailer's profit when there is no space exchange.

3.2. Optimal Prices When $\bar{u}_i \leq c_i + t$

When $\bar{u}_i \leq c_i + t$ ($i=1,2$), we found in Lemma 2 that, if there is no capacity constraint, retailer i 's profit is maximized when $p_{ii} = p_{ij} = (\bar{u}_i + c_i)/2$ ($j = 1,2, j \neq i$) and the corresponding demands in two stores are $D_{ii} = D_{ij} = B_i(\bar{u}_i - c_i)/(2t)$. We now investigate the retailer's optimal pricing decisions given his host space and guest space.

LEMMA 5. Suppose that retailer i 's ($i = 1,2$) host space and guest space are given as $S_{ii} \in [0, S_i]$ and $S_{ji} \in [0, S_j]$, for $j = 1,2, j \neq i$. If $\bar{u}_i \leq c_i + t$, retailer i 's optimal prices in two stores are found as $p_{ii}^* = \max[(\bar{u}_i + c_i)/2, \bar{u}_i - tS_{ii}/B_i]$ and $p_{ij}^* = \max[(\bar{u}_i + c_i)/2, \bar{u}_i - tS_{ji}/B_i]$. If $S_i \geq B_i(\bar{u}_i - t)/(2t)$, then retailer i 's optimal prices p_{ii}^* and p_{ij}^* under the space-exchange strategy are both higher than his optimal price p_i^* when two retailers do not exchange shelf space. However, if $S_i < B_i(\bar{u}_i - t)/(2t)$, then p_{ii}^* is always greater than p_i^* ; but p_{ij}^* may or may not be greater than p_i^* . We find that $p_{ij}^* > p_i^*$ when $S_{ji} \leq S_i$.

Lemma 5 gives us retailer i 's pricing decisions in two stores, when the product i consumer residing at the location of a retailer's store cannot enjoy a positive net utility if he or she decides to buy in the other retailer's store (i.e., $\bar{u}_i \leq c_i + t$). As Lemma 5 implies, the retailer should increase his price in his own store when he cooperates with the other retailer for the space-exchange strategy. However, after retailer i also operates in retailer j 's store using the guest space S_{ji} , his price for product i in retailer j 's store may be lower than that in retailer i 's own store before the

Table 3 Retailer i 's Maximum Profit When $\bar{u}_i > c_i + t$

Conditions		Retailer's maximum profit	
$S_{ii} + S_{ji} \geq B_i$	$S_{ii} \geq B_i/2, S_{ji} \geq B_i/2$		$\pi_i^1 \equiv B_i(\bar{u}_i - c_i - t/2)$
	$S_{ii} < \frac{B_i}{2}$	$S_{ji} \geq B_i(1 - \frac{\bar{u}_i - c_i}{2t})$	$\pi_i^2 \equiv B_i(\bar{u}_i - c_i - t) + 2tS_{ji}(1 - S_{ii}/B_i)$
	$S_{ji} > \frac{B_i}{2}$	$S_{ii} \leq B_i(1 - \frac{\bar{u}_i - c_i}{2t})$	$\pi_i^3 \equiv B_i(\bar{u}_i - c_i)^2/(4t) + (\bar{u}_i - tS_{ii}/B_i - c_i)S_{ji}$
	$S_{ii} > \frac{B_i}{2}$	$S_{ji} \geq B_i(1 - \frac{\bar{u}_i - c_i}{2t})$	$\pi_i^4 \equiv B_i(\bar{u}_i - c_i - t) + 2tS_{ji}(1 - S_{ii}/B_i)$
	$S_{ji} < \frac{B_i}{2}$	$S_{ii} \leq B_i(1 - \frac{\bar{u}_i - c_i}{2t})$	$\pi_i^5 \equiv B_i(\bar{u}_i - c_i)^2/(4t) + (\bar{u}_i - tS_{ii}/B_i - c_i)S_{ji}$
	$S_{ii} \leq B_i \frac{\bar{u}_i - c_i}{2t}$	$S_{ji} \leq B_i \frac{\bar{u}_i - c_i}{2t}$	$\pi_i^6 \equiv (\bar{u}_i - tS_{ii}/B_i - c_i)S_{ii} + (\bar{u}_i - tS_{ji}/B_i - c_i)S_{ji}$
$S_{ii} + S_{ji} < B_i$	$S_{ii} \geq B_i \frac{\bar{u}_i - c_i}{2t}$	$S_{ji} \leq B_i \frac{\bar{u}_i - c_i}{2t}$	$\pi_i^7 \equiv B_i(\bar{u}_i - c_i)^2/(4t) + (\bar{u}_i - tS_{ji}/B_i - c_i)S_{ji}$
	$S_{ii} \leq B_i \frac{\bar{u}_i - c_i}{2t}$	$S_{ji} \geq B_i \frac{\bar{u}_i - c_i}{2t}$	$\pi_i^8 \equiv B_i(\bar{u}_i - c_i)^2/(4t) + (\bar{u}_i - tS_{ii}/B_i - c_i)S_{ii}$

space exchange. More specifically, if the total space S_i in retailer i 's store is sufficiently large [i.e., $S_i \geq B_i(\bar{u}_i - t)/(2t)$], then, no matter what the guest space S_{ji} —that is allocated by retailer j to retailer i —is, retailer i should always set the price p_{ij} higher than p_i^* . Otherwise, if retailer i cannot use the total space S_i in his own store to serve half of B_i product i consumers, i.e., $S_i < B_i(\bar{u}_i - c_i)/(2t)$, then the retailer may or may not set a price higher than p_i^* , which depends on the value of the guest space S_{ji} . If the guest space is larger than the total space S_i in retailer i 's own store, then retailer i may choose a price lower than p_i^* to entice more consumers to buy product i in retailer j 's store, because the space in the retailer's own store is very small. Otherwise, if the guest space S_{ji} is also very small (i.e., $S_{ji} < S_1$), then retailer i is unable to serve all (or even, most of) product i consumers, and should thus increase the prices in two stores to improve his profit.

Lemmas 4 and 5 indicates the comparison between retailer i 's optimal prices with and without the space exchange, for the case that $\bar{u}_i > c_i + t$ and the case that $\bar{u}_i \leq c_i + t$, respectively. Using these results, we reach a conclusion regarding the impacts of the space-exchange strategy on the retail prices, as given in the following proposition.

PROPOSITION 1. *When retailers 1 and 2 implement the space-exchange strategy, each retailer's prices for his products in two stores are higher than the retailer's price in his own store when two retailers do not exchange shelf space.*

PROOF. A proof for this proposition and our proofs for all subsequent propositions are provided in Appendix S2. $\square$

The above proposition indicates that, if two retailers decide to exchange shelf space, then they should raise their retail prices. Using Lemmas 3 and 5, we can also find the following result regarding each retailer's two prices in two stores.

PROPOSITION 2. *Retailer i ($i = 1, 2$) may determine different prices for product i in two stores. That is, after two retailers exchange shelf space, the retail prices of the same products at two stores may not be identical.*

We also note from Lemmas 3 and 5 that retailer i 's optimal pricing decisions when $\bar{u}_i \leq c_i + t$ are the same as those when $\bar{u}_i > c_i + t$ and $S_{ii} + S_{ji} < B_i$. Thus, the retailer's possible maximum profits when $\bar{u}_i \leq c_i + t$ should include those when $S_{ii} + S_{ji} < B_i$ in Table 3. Similar to Corollary 1, we find that, when $\bar{u}_i \leq c_i + t$, π_i^k ($k = 7, 8$)—as given in Table 3—it is

greater than retailer i 's profit when two retailers do not exchange shelf space; however, π_i^6 in Table 3 may be smaller than the retailer's profit with no space exchange, which depends on the values of S_{ii} and S_{ji} .

In addition to π_i^k ($k = 6, 7, 8$) in Table 3, retailer i may achieve the maximum profit $\pi_i^9 \equiv B_i(\bar{u}_i - c_i)^2/(2t)$, which occurs when $\bar{u}_i \leq c_i + 2tS_{ii}/B_i$ and $\bar{u}_1 \leq c_1 + 2tS_{21}/B_1$. Note that π_i^9 is not considered for the case that $S_{ii} + S_{ji} < B_i$ in Table 3, because the conditions that $\bar{u}_i \leq c_i + 2tS_{ii}/B_i$ and $\bar{u}_i \leq c_i + 2tS_{ji}/B_i$ imply that $S_{ii} + S_{ji} \geq B_i$, which is contrary to the case that $S_{ii} + S_{ji} < B_i$.

Similar to our previous discussion for the case that $\bar{u}_i > c_i + t$, we find that, if retailer i 's host space and guest space are both small, then the retailer may not achieve a higher profit from implementing the space-exchange strategy and may thus lose the incentive to cooperate with retailer j . On the other hand, if the retailer has a sufficiently large space in his own store and/or retailer j 's store, then he should obtain a profit that is higher than the profit with no space exchange. That is, to entice retailer i to exchange his space with retailer j , retailer j may need to allocate a sufficiently large space to retailer i .

4. Nash Equilibrium Space-Allocation Decisions

We now investigate the optimal allocation of each store's shelf space between two retailers in the equilibrium. That is, we determine the optimal values of S_{ii} and S_{ij} in retailer i 's store where $S_{ii} + S_{ij} = S_i$, for $i, j = 1, 2$ and $i \neq j$. As the total space in each store (i.e., S_i) is given, retailer i only needs to determine the value of S_{ii} , and allocates the remaining space $S_{ij} = S_i - S_{ii}$ to retailer j . To find the optimal space decision, each retailer should maximize the sum of his profits generated in two stores. Thus, the space allocation problem can be naturally regarded as a "simultaneous-move" non-cooperative game, and the two retailers' optimal space allocation decisions should be characterized by Nash equilibrium.

To solve the non-cooperative game and find the Nash equilibrium, we need to first analyze each retailer's best response—i.e., the optimal space decision for a given space allocation decision of the other retailer. Next, we begin by finding retailer i 's best space allocation decision for his own store, assuming that retailer j decides to retain the space S_{jj} and allocate the space S_{ji} to retailer i .

4.1. The Best-Response Analysis

To implement the space-exchange strategy, retailer i ($i = 1, 2$) uses the guest space S_{ji} —given by retailer j ($j = 1, 2, j \neq i$)—to serve some or all product i consumers, and determines his host space $S_{ii} \in [0, S_i]$ and

allocate the space $S_{ij} = S_i - S_{ii}$ to retailer j . Thus, to find the best response to retailer j 's space allocation decision, retailer i should find the optimal host space that maximizes his own profit. As discussed in section 3, we calculate retailer i 's maximum profit, given the retailer's host space and guest space, as shown in Table 3.

Next, we assume that the guest space $S_{ji} \in [0, S_j]$ is given, and use our results in Table 3 to find the optimal host space S_{ii} (best response) for retailer i . As our analysis in section 3 indicates that retailer i 's optimal pricing decisions depend on the comparison between $\bar{u}_i$ and $c_i + t$, we consider the retailer's optimal space allocation decision for the two cases: $\bar{u}_i > c_i + t$ and $\bar{u}_i \leq c_i + t$.

4.1.1. The Best Response When $\bar{u}_i > c_i + t$. We now determine retailer i 's best space allocation decision, when the product i consumer residing at the site of retailer j 's store can gain a positive net utility if he or she visits retailer i 's store to buy product i (i.e., $\bar{u}_i > c_i + t$).

LEMMA 6. *When $\bar{u}_i > c_i + t$, retailer i 's optimal space allocation decision depends on the values of S_{ji} and the total space S_i in his own store, as shown in Table 4.*

From the above lemma, we learn that, if either the total space S_i in retailer i 's own store or the retailer's guest space S_{ji} (allocated by retailer j) cannot be used to serve a half of B_i product 1 consumers, then retailer i should make his space allocation decision to serve a part (rather than all) of B_i consumers. Otherwise, if both S_i and S_{ji} are large enough to serve a half of product i consumers, then the retailer should determine his host space such that all consumers will buy in two stores. This may reflect the fact that retailer i intends to serve all product i consumers using his space in two stores. Thus, if retailer j allocates a sufficiently large space to retailer i and the total space S_i in retailer i 's store is also sufficiently large, then retailer i should decide to retain a host space that is large enough to assure that he can serve all consumers in two stores.

However, if retailer i cannot serve $B_i/2$ consumers in his guest space S_{ji} , then he should not retain a large host space to serve all consumers, which may be justifi-

fied as follows: When the guest space is so small that less than a half of consumers are willing to buy, retailer i should have to use his host space to serve more than a half of B_i consumers if he intends to serve all consumers in two stores. However, to sell more than $B_i/2$ units of product i in retailer i 's own store, the retailer has to reduce his retail price to a low level, which may thus reduce his total profit. Similarly, if S_i is small, then retailer i should accept a small guest space from retailer j and should not serve all consumers.

4.1.2. The Best Response When $\bar{u}_i \leq c_i + t$. From Lemmas 3 and 5, we find that if $\bar{u}_i \leq c_i + t$, then retailer i 's optimal prices in two stores are the same as those when $\bar{u}_i > c_i + t$ and $S_{ii} + S_{ji} < B_i$. Thus, the retailer's maximum profit when $\bar{u}_i \leq c_i + t$ is the same as that when $S_{ii} + S_{ji} < B_i$ in Table 3.

LEMMA 7. *If $\bar{u}_i \leq c_i + t$ ($i = 1, 2$), then retailer i 's best-response space allocation decision is the same as that when $S_i + S_{ji} < B_i$ in last row of Table 4.*

As the above lemma indicates, retailer i ($i = 1, 2$) should make his space allocation decision to serve a part of B_i product i consumers if the product i consumer at the site of retailer j 's ($j = 1, 2, j \neq i$) store cannot enjoy a positive net utility when he or she buys in retailer i 's store, i.e., $\bar{u}_i \leq c_i + t$. For this case, if two retailers do not implement the space-exchange strategy, then some consumers who are closer to the site of retailer j do not buy product i . After the space exchange, retailer i may need to utilize his guest space to serve those consumers (who do not buy before the space exchange). Note that, in the linear city, a half of total product i consumers (i.e., B_i consumers) are closer to retailer j 's store, and they could prefer to buy in retailer j 's rather than retailer i 's store. This means that, as a consequence of the space exchange, retailer i may serve fewer consumers in his store and may thus raise the retail price in his own store to increase his profit margin. In addition, to assure the retailer's profit in retailer j 's store, the retailer should not reduce his price for product i in retailer j 's store to a low level, which may discourage some consumers from buying product i . Therefore, to maximize retailer i 's total profit in two stores, the retailer may make

Table 4 Retailer i 's Best-Response Space Decision When $\bar{u}_i > c_i + t$.

Conditions	Optimal space decision
$S_i + S_{ji} \geq B_i$	$S_{ii}^* = \min[S_i, B_i/2], S_{ij}^* = [0, S_i - B_i/2]^+.$ (All consumers buy.)
$S_{ji} \geq B_i/2$	$S_{ii}^* = S_i$ and $S_{ij}^* = 0.$ (Some consumers may not buy.)
$S_i < B_i[1 - (\bar{u}_i - c_i)/(2t)]$	$S_{ii}^* = B_i - S_{ji}, S_{ij}^* = S_i - B_i + S_{ji}.$ (All consumers buy.)
$B_i[1 - (\bar{u}_i - c_i)/(2t)] \leq S_{ji} < B_i/2$	$S_{ii}^* = B_i(\bar{u}_i - c_i)/(2t), S_{ij}^* = S_i - B_i(\bar{u}_i - c_i)/(2t).$ (Some consumers may not buy.)
$S_{ji} < B_i[1 - (\bar{u}_i - c_i)/(2t)]$	$S_{ii}^* = \min[B_i(\bar{u}_i - c_i)/(2t), S_i], S_{ij}^* = [0, S_i - B_i(\bar{u}_i - c_i)/(2t)]^+.$ (Some consumers may not buy.)
$S_i + S_{ji} < B_i$	

his pricing and space allocation decisions to only serve a part of B_i consumers.

4.2. Nash Equilibrium

We use our above best-response analysis for two retailers to find the Nash equilibrium (S_{ii}^N, S_{jj}^N) ($i, j = 1, 2, i \neq j$) for the non-cooperative game. Note that the guest space S_{ij}^N and S_{ji}^N in two stores are computed as $S_{ij}^N = S_i - S_{ii}^N$ and $S_{ji}^N = S_j - S_{jj}^N$; and after each retailer makes his or her optimal space decision, the retailer's optimal price is correspondingly determined using our results in section 3.

We find from our best-response analysis that in some cases, a retailer may allocate zero space to the other retailer. For example, if $\bar{u}_i - c_i > t$, $S_i + S_{ji} \geq B_i$, $S_{ji} \geq B_i/2$, and $B_i[1 - (\bar{u}_i - c_i)/(2t)] \leq S_i \leq B_i/2$, then retailer i should not allocate any space to retailer j , as indicated by Lemma 6. Naturally, to implement the space-exchange strategy, each retailer must allocate some non-zero space to the other retailer. Therefore, if a retailer does not allocate any space to the other retailer, then two retailers should not consider the space-exchange strategy, but instead operate with no space exchange.

PROPOSITION 3. *In the Nash equilibrium, retailers 1 and 2 should decide to implement the space-exchange strategy if and only if $S_i > \max[B_i[1 - (\bar{u}_i - c_i)/(2t)], B_i(\bar{u}_i - c_i)/(2t)]$, for $i = 1, 2$.*

In the above proposition, we note that, for retailer i ($i = 1, 2$), either $[1 - (\bar{u}_i - c_i)/(2t)]$ or $(\bar{u}_i - c_i)/(2t)$ must be $\geq 1/2$. That is, for the “simultaneous-mover” game, the total shelf space S_i in retailer i 's own store must be large enough to serve more than a half of B_i product i consumers to let retailer i have an incentive for the space exchange with retailer j ($j = 1, 2, j \neq i$). Thus, two retailers should have sufficient shelf space in their own stores to implement the space-exchange strategy. Otherwise, they may have no incentive for the space exchange in the game.

One may note that two retailers with small shelf space could also consider the space-exchange strategy. For example, suppose that retailer i ($i = 1, 2$) can stock only two units of product i in his store before the space exchange, i.e., $S_i = 2$, for $i = 1, 2$. When two retailers do not exchange shelf space, retailer i would set his price such that the two consumers who are the closest to the retailer along the Hotelling line would find it worthwhile to buy product i . If two retailers exchange shelf space, then they may raise their prices without losing any consumers, and their profits could thus be higher than those in the “no space exchange” case. This differs from Proposition 3, which is justified as follows: Proposition 3 holds when two retailers

make their decisions in the *non-cooperative* game, whereas the above discussion is based on the assumption that two retailers jointly make their decisions in the *cooperative* setting. For a detailed discussion, see Appendix S5. Note that we use the Hotelling model to analyze the space-exchange problem; thus, Proposition 3 applies to the non-cooperative setting.

As two retailers decide to exchange shelf space in the “simultaneous-move” game if and only if the non-zero space allocation decisions exist in Nash equilibrium, we next analyze our non-cooperative space-exchange game to find Nash equilibrium under the condition in Proposition 2. We learn from our previous analysis that two retailers make their pricing and space decisions according to whether or not all consumers for each product can enjoy a positive net utility from buying at each end point (i.e., the site of each retailer's store) of the linear city. Accordingly, we should compare $\bar{u}_i$ and $c_i + t$ for retailer i to compute the Nash equilibrium for the space-exchange problem. Hence, for our game analysis, we need to consider the following three cases: (i) $\bar{u}_i \leq c_i + t$, for $i = 1, 2$; (ii) $\bar{u}_i > c_i + t$ and $\bar{u}_j \leq c_j + t$, for $i, j = 1, 2, i \neq j$; and (iii) $\bar{u}_i > c_i + t$, for $i = 1, 2$.

Our best-response analysis indicates that, when $\bar{u}_i > c_i + t$, retailer i has a number of different optimal space decisions dependent on the space in his own store and his guest space. Therefore, for Case (iii), there should be a number of possible Nash equilibria, which depend on the total space in each retailer's store. To facilitate our discussion, we first consider Case (i), and find the corresponding Nash equilibrium. This is then followed by our discussion of the remaining two cases.

4.2.1. Nash Equilibrium When $\bar{u}_i \leq c_i + t$ ($i = 1, 2$). Using our best-response analysis in section 4.1, we now solve the two-person non-cooperative game to find the Nash equilibrium for retailers 1 and 2.

LEMMA 8. *If $\bar{u}_i \leq c_i + t$ ($i = 1, 2$), then Nash equilibrium uniquely exists as $S_{ii}^N = B_i(\bar{u}_i - c_i)/(2t)$ and $S_{jj}^N = B_j(\bar{u}_j - c_j)/(2t)$, for $j = 1, 2$ and $j \neq i$. Retailer i should allocate the space $S_{ij}^N = S_i - B_i(\bar{u}_i - c_i)/(2t) > 0$ to retailer j , who allocates the space $S_{ji}^N = S_j - B_j(\bar{u}_j - c_j)/(2t) > 0$ to retailer i .*

To implement the equilibrium space decision, retailer i should determine the corresponding prices for product i in his own store and retailer j 's store as $p_{ii}^N = (\bar{u}_i + c_i)/2$ and $p_{ij}^N = \bar{u}_i - tS_{ij}^N/B_i$, respectively.

From the above lemma, we find that, to implement the space-exchange strategy, each retailer allocates a non-zero space to the other retailer. As indicated by Lemma 7, retailer i ($i = 1, 2$) should *fully* use his guest

space S_{ji}^N ($j = 1, 2$ and $j \neq i$) to serve some product i consumers who are closer to retailer j 's store, respectively. However, according to our best-response analysis, we note that two retailers do not serve all of their consumers, when they choose the Nash equilibrium. Nevertheless, the two retailers' prices in both host space and guest space are higher than their prices determined when there is no space exchange, as shown in Lemma 5.

We also learn from Lemma 8 that each retailer may or may not set an identical price in two stores. If the space allocated by retailer j to retailer i is $S_{ji}^N = B_i(\bar{u}_i - c_i)/(2t)$, then retailer i 's Nash equilibrium prices in two stores (i.e., p_{ii}^N and p_{ij}^N) will be identical. As $S_{ji}^N = S_j - B_j(\bar{u}_j - c_j)/(2t)$, we find that $p_{ii}^N = p_{ij}^N$ if $S_j = B_i(\bar{u}_i - c_i)/(2t) + B_j(\bar{u}_j - c_j)/(2t)$. That is, if S_j equals the space that is needed to stock $B_i(\bar{u}_i - c_i)/(2t)$ units of product i and $B_j(\bar{u}_j - c_j)/(2t)$ units of product j , then retailer i should set an identical price $(\bar{u}_i + c_i)/2$ for product i in two stores. Otherwise, retailer i should determine different prices.

Moreover, two retailers can achieve higher profits compared with those before they exchange shelf space, which means that they should have incentives to cooperate with the space-exchange strategy.

4.2.2. Nash Equilibrium When $\bar{u}_i > c_i + t$ and $\bar{u}_j \leq c_j + t$ ($i, j = 1, 2$ and $i \neq j$). We now consider the case where all product i ($i = 1, 2$) consumers can achieve a positive net utility from buying in two stores (i.e., $\bar{u}_i > c_i + t$) but some product j ($j = 1, 2$ and $j \neq i$) consumers (e.g., the consumer residing at the site of a retailer's store) cannot draw a positive net utility from their purchases in a store (i.e., $\bar{u}_j \leq c_j + t$).

LEMMA 9. When $\bar{u}_i > c_i + t$ and $\bar{u}_j \leq c_j + t$, for $i, j = 1, 2$ and $i \neq j$, there exists a unique Nash equilibrium, which depends on the value of S_j , as given in Table 5.

Similar to our analysis for Case (i) $\bar{u}_i \leq c_i + t$ ($i = 1, 2$), we can use Lemma 3 and 5 to compute two retailers' corresponding optimal prices for each of three possible Nash equilibria in Table 5. We find that each retailer's prices in two stores are both *higher* than the retailer's price before the space exchange. This means that implementing the space-exchange strategy raises the retail prices. From Lemma 9 we find that, if

not all product j ($j = 1, 2$ and $j \neq i$) consumers can enjoy a positive net utility, then the total space S_j in retailer j 's store impacts two retailers' Nash equilibrium decisions. Moreover, whatever the value of S_j is, retailer j 's Nash equilibrium space is always determined as $B_j(\bar{u}_j - c_j)/(2t)$. Using Table 5, we use Figure 2 to show retailer i 's Nash equilibrium space decision S_{ii}^N as a function of S_j .

If S_j is larger than or equal to $B_i/2 + B_j(\bar{u}_j - c_j)/(2t)$, then retailer j allocates a sufficiently large guest space to retailer i who can then serve a half of product i consumers in retailer j 's store. According to Figure 2, we find that retailer i should retain his host space S_{ii}^N to serve the other half of his consumers to serve all B_i consumers. However, when S_j is reduced to a value between $B_i[1 - (\bar{u}_i - c_i)/(2t)] + B_j(\bar{u}_j - c_j)/(2t)$ and $B_i/2 + B_j(\bar{u}_j - c_j)/(2t)$, we learn from Figure 2 that retailer i should increase his host space mainly because of the following fact: When S_j decreases, retailer j allocates to retailer i a smaller guest space where retailer i cannot serve a half of product i consumers. To fulfill the demands of all B_i consumers in the linear city, retailer i should keep a sufficiently large host space such that all of the consumers who do not decide to buy in retailer j 's store are willing to shop in retailer i 's store.

When S_j is smaller than $B_i[1 - (\bar{u}_i - c_i)/(2t)] + B_j(\bar{u}_j - c_j)/(2t)$, retailer i will obtain a very small guest space and have to serve a small number of product i consumers in retailer j 's store. If retailer i still hopes to fulfill all consumers' demands, then the retailer should keep a large host space to serve most of his consumers. To assure that most consumers

Figure 2 The Impact of the Space S_j ($j = 1, 2$) on Retailer i 's ($i = 1, 2$, $i \neq j$) Nash Equilibrium S_{ii}^N When $\bar{u}_i > c_i + t$ and $\bar{u}_j \leq c_j + t$

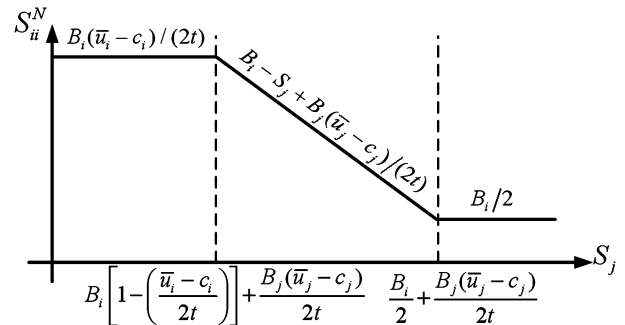


Table 5 Nash Equilibrium When $\bar{u}_i > c_i + t$ and $\bar{u}_j \leq c_j + t$, for $i, j = 1, 2$ and $i \neq j$

Conditions	Nash equilibrium
$S_j \geq B_i/2 + B_j(\bar{u}_j - c_j)/(2t)$	$S_{ii}^N = B_i/2, S_{jj}^N = B_j(\bar{u}_j - c_j)/(2t)$
$B_i[1 - (\bar{u}_i - c_i)/(2t)] + B_j(\bar{u}_j - c_j)/(2t) \leq S_j \leq B_i/2 + B_j(\bar{u}_j - c_j)/(2t)$	$S_{ii}^N = B_i - S_j + B_j(\bar{u}_j - c_j)/(2t), S_{jj}^N = B_j(\bar{u}_j - c_j)/(2t)$
$S_j \leq B_i[1 - (\bar{u}_i - c_i)/(2t)] + B_j(\bar{u}_j - c_j)/(2t)$	$S_{ii}^N = B_i(\bar{u}_i - c_i)/(2t), S_{jj}^N = B_j(\bar{u}_j - c_j)/(2t)$

(especially those closer to retailer j 's store) are willing to buy in retailer i 's store, retailer i should set a sufficiently low retail price for product i in his host space, which, however, results in a low profit. Therefore, if S_j is significantly small, then retailer i should determine his host space as $S_{ii}^N = B_i(\bar{u}_i - c_i)/(2t)$ to guarantee his profit; and as a result, some product i consumers will decide not to buy.

4.2.3. Nash Equilibrium When $\bar{u}_i > c_i + t$ ($i = 1, 2$). We now solve our non-cooperative game for Case (iii) where both product 1 consumers and product 2 consumers can gain a positive net utility from buying in any store, i.e., $\bar{u}_i > c_i + t$, for $i = 1, 2$. The analysis for this case is much more complicated than the above two cases, and there are many possible Nash equilibria. Thus, we only show the existence of Nash equilibrium in the following theorem, but the specific Nash equilibria are provided in Appendix S6.

LEMMA 10. *When $\bar{u}_i > c_i + t$ ($i = 1, 2$), then the corresponding Nash equilibrium (S_{ii}^N, S_{jj}^N) ($j = 1, 2$ and $j \neq i$) must exist, but it may or may not be unique. More specifically, if the total shelf space of two stores is not the same as that needed to exactly serve two retailers' consumers, then Nash equilibrium for the game uniquely exists. Otherwise, the Nash equilibrium may not be unique, which depends on consumers' utilities and trip costs as well as the total available shelf space in each retailer's store. All possible Nash equilibria are given in Table S6 (see Appendix S6).*

Similar to our analysis for the above cases, we can use Lemma 3 to compute two retailers' corresponding optimal prices for each possible Nash equilibrium given in Table S6. Moreover, we find from Lemma 3 that each retailer's optimal prices in both the host space and the guest space are *higher* than the retailer's price when two retailers do not implement the space-exchange strategy. That is, such a strategy induces two retailers to increase their prices.

5. Further Discussions

In the preceding sections, we analyzed the space-exchange problem and found the Nash equilibrium pricing and space-allocation decisions. We now provide a further discussion on possible changes of our major results in the following two settings: We first consider a more realistic case where there exists at least one "common" consumer who buys both products 1 and 2, and then investigate whether or not our results will change in the setting where a retailer's fixed cost of opening and staffing a new store is considered.

5.1. Presence of Common Consumers

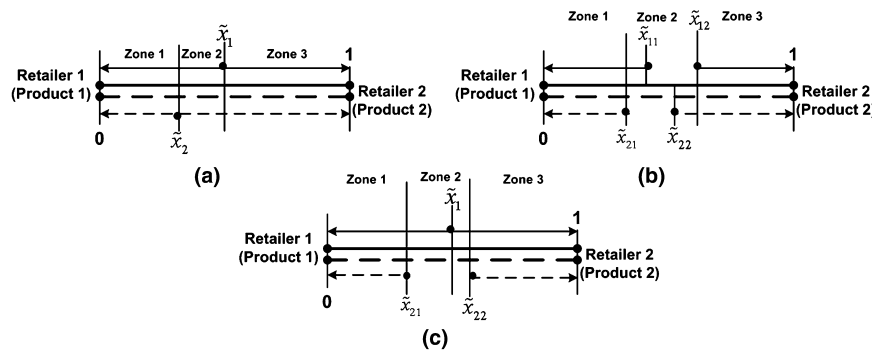
In our model, the set of product 1 consumers and the set of product 2 consumers are assumed to be disjoint. This means that, at any point along the Hotelling line, the product 1 consumer is different from the product 2 consumer. Such an assumption may be applicable to the space-exchange problem to some extent for the following reason: As indicated by the practice of Waitrose and Boots and also by that of Tim Hortons and Cold Stone Creamery, the space-exchange strategy applies *only* when the cooperating retailers' products are neither substitutable nor complementary, e.g., Waitrose's food vs. Boots's healthcare products; and, Tim Hortons's doughnuts vs. Cold Stone Creamery's ice cream. Hence, it should be unusual for any common consumer along the line to *intentionally* buy both products at the same time. For this case, we could regard the common consumer as a "product 1 consumer" and also a "product 2 consumer," who are independent of each other; and then, our existing model could be still used to analyze the space-exchange problem.

Despite the above argument, in reality, there may still exist some common consumer(s) who intend to buy both products concurrently. However, if we relax our assumption on the dependence of the product 1 consumers and the product 2 consumers, then our model may become intractably complicated and we could not draw any meaningful analytical insights. Therefore, we do not incorporate such common consumers into our model, but subsequently discuss how our major results would possibly change when a common consumer (who intends to buy both products concurrently) exists.

We learn from our analysis in sections 3 and 4 that, in the Nash equilibrium, retailer i ($i = 1, 2$) may serve all of B_i consumers or may serve only a part of those consumers. Next, we provide our discussion for three cases: (a) both retailers i and j ($j = 1, 2$ and $j \neq i$) serve all of their consumers; (b) both retailers i and j do not serve all of their consumers; and (c) retailer i serves all of B_i consumers, but retailer j does not serve all of B_j consumers. For each case, we discuss a representative situation, as shown in Figure 3, where (a), (b), and (c) represent Cases (a), (b), and (c), respectively. (Our discussions on other situations in each case are similar to our discussion for the representative situation.)

We begin by discussing the impacts of the presence of a common consumer in Case (a), which corresponds to Figure (3a) where the product i ($i = 1, 2$) consumers located at the left of $\tilde{x}_i$ buy in retailer 1's store, and those consumers at the right of $\tilde{x}_i$ buy in retailer 2's store. As Figure (3a) indicates, the product 1 and the product 2 consumers in zone 1 visit retailer 1's store to buy products 1 and 2, respectively, when there is no common consumer. If we assume that

Figure 3 The Impacts of the Presence of a Common Consumer on Major Results for Three Representative Situations



there is a common consumer at a point in zone 1, then the consumer will still buy both products in retailer 1's store. That is, for Case (a), the presence of common consumers in zone 1 would not increase the demands for two products in each store; thus, it may not result in any change in two retailers' pricing and space-allocation decisions. Similarly, any common consumer in zone 3 will decide to buy two products in retailer 2's store. This does not increase or decrease the demands faced by two retailers in each store, and would not change two retailers' decisions.

If a common consumer is located in zone 2, then the total demand for each product in two stores should be unchanged, but the demands faced by two retailers in each store may differ from those with no common consumer. Specifically, if there is no common consumer in zone 2, then the product 1 and the product 2 consumers will buy in different stores. However, if a common consumer in zone 2 is closer to $\tilde{x}_2$, then he or she may be likely to buy both products in retailer 1's store. As a result, compared with the "no common consumer" case, the demand for product 2 in retailer 1's store is increased by 1 unit and that in retailer 2's store is decreased by 1 unit, whereas the demand for product 1 in each store is not changed. Even though retailer 2 needs to sell one more unit in retailer 1's store, retailer 1 is unlikely to allocate one more space to retailer 2, because retailer 1 should keep his host space to serve existing customers. If two retailers have already used the total space S_1 , then retailer 2 may respond by increasing his price p_{21} for product 2 in retailer 1's store, but decreasing his price p_{22} for product 2 in his own store to "move" a consumer from retailer 1's store to retailer 2's store. On the other hand, if there is an excess space in retailer 1's store, then retailer 2 should take over one more unit of the space to satisfy the demand by the common consumer; as a result, two retailers do not change their pricing decisions.

Next, we discuss the impacts of the presence of a common consumer in Case (b), which corresponds to

Figure (3b) where the product i ($i = 1, 2$) consumers residing at the left of $\tilde{x}_{i1}$ and those at the right of $\tilde{x}_{i2}$ buy in retailer 1's and retailer 2's stores, respectively. Using our arguments for Case (a), a common consumer in zones 1 and 3 would have no impact on two retailers' decisions and the presence of a common consumer in zone 2 would lead to an increase in the demand faced by one or two retailers at a store. The retailers may change their prices at two stores if there is no excess space at the store where the demand rises, or may keep the prices unchanged and use the excess space otherwise. Similarly, we find that, for Case (c), there would be no change if a common consumer is in zones 1 and 3, but the existence of a common consumer in zone 2 would result in the price changes at two stores when there is no excess space. Summarizing our above discussion, we draw the implications as given in the following remark.

REMARK 3. If a consumer intends to buy both products 1 and 2, then the demand faced by one or two retailers at a store may be increased and two retailers may respond by changing their pricing and space-allocation decisions. Specifically, the presence of a common consumer close to a store is unlikely to change two retailers' decisions. However, if a common consumer resides in a middle point between two stores, then one or two retailers may face an increasing demand at a store, thereby increasing their prices if there is no excess space at the store or keeping the prices unchanged, but using the excess space. As a result, if a common consumer exists, then two retailers' profits could be increased. Moreover, our above discussion also implies that Propositions 1, 2, and 3 should hold in the presence of common consumers.

5.2. Presence of Fixed Costs

In actual practice, two retailers (e.g., Waitrose and Boots, Tim Hortons and Cold Stone Creamery) may be willing to exchange shelf space instead of opening

their own new stores. A motivation for two retailers to exchange shelf space would be mainly attributed to the fact that each retailer incurs a fixed cost in opening and staffing a new store, but does not pay for such a cost in exchanging his shelf space with the other retailer. We now examine whether or not our major results would change if we consider fixed costs in our Hotelling model. Suppose that retailer i ($i = 1, 2$) will decide to (i) open a new store at retailer j 's site ($j = 1, 2$ and $j \neq i$), or (ii) exchange shelf space with retailer j . Each retailer should choose one from the two options (i) and (ii).

Noting that two retailers' fixed costs are independent of their pricing and space-allocation decisions, we find that, if each retailer's store is sufficiently large, then incorporating such costs into our model should not change two retailers' decisions, and exchanging shelf space should result in a higher profit for each retailer compared with opening a new store. Otherwise, if a retailer's store is small, then the retailer cannot allot a sufficiently large space to the other retailer, who may then respond by opening a new store instead of exchanging shelf space. Such a result is in agreement with Proposition 3, which indicates that two retailers should decide to implement the space-exchange strategy in Nash equilibrium, if and only if each retailer's total shelf space is large enough to serve more than a half of his consumers. In section 4, we perform our game analysis, assuming that each retailer has a sufficiently large store. Such an assumption is compatible with the practice that the retailers exchanging shelf space include, e.g., Waitrose, Boots, Tim Hortons, and Cold Stone Creamery. It thus follows that our results in this article do not change if we consider each retailer's fixed cost of opening and staffing a new store.

6. Summary and Concluding Remarks

This article is motivated by the practice of Waitrose and Boots (and also, Tim Hortons and Cold Stone Creamery) where these retailers exchange shelf space to increase their profits. We use the Hotelling model to analyze a two-retailer problem. Before the space exchange, each consumer can buy only in one store; however, after two retailers implement the space-exchange strategy, the consumer can access each retailer's product in two stores and thus visit a store closer to the consumer's location to buy.

We first assume that two retailers do not exchange shelf space, and maximize each retailer's profit to find the optimal price for the retailer's product in his own store. Then, we determine each retailer's optimal prices in two stores, given his host space and guest space under the space-exchange strategy, and find that the space-dependent prices are impacted by

whether or not all of the retailer's consumers can enjoy a positive net utility from buying in any store.

Using the optimal space-dependent price, we consider a non-cooperative game where each retailer makes the space allocation decision for his own store to maximize the total profit in two stores. We show that two retailers should decide to implement the space-exchange strategy in the game, if and only if the total space in each retailer's store is large enough to serve more than a half of the retailer's consumers. Nash equilibrium for the game may or may not uniquely exist, depending on consumers' utilities and trip costs as well as the total space in each store. We also find that, in the Nash equilibrium, each retailer's prices in two stores may or may not be identical, but they are both higher than the retailer's price before the space exchange, and two retailers' profits are higher than those before they implement the space-exchange strategy. We also discuss possible changes of our major results when there exists a common consumer who buys both products 1 and 2, and those when a retailer's fixed cost for opening and staffing a new store is considered.

As we discuss in section 1, the informational advantage and the risk reduction should be the two main advantages of the space-exchange strategy. In this article, we focus on consumers' increased choices and reduced trip costs, which should be the major advantage for the strategy. The analysis of the informational advantage and the risk reduction for the space-exchange problem would be a future research direction.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1: Proofs of Lemmas.

Appendix S2: Proofs of Propositions.

Appendix S3: Detailed Discussion for Remark 1.

Appendix S4: Proof of Corollary 1.

Appendix S5: A Further Discussion on the Result in Proposition 3.

Appendix S6: Nash Equilibrium When $\bar{u}_i > c_1 + t$ ($i = 1, 2$).

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