Disclosing Delivery Performance Information when Consumers are Sensitive to Promised Delivery Time, Delivery Reliability, and Price

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Abstract

Problem definition: We investigate how the characteristics of consumers and a service firm influence the firm's optimal pricing and promised delivery-time decisions as well as the optimal investment in the quality of delivery reliability information available to consumers. *Methodology/Results*: We use utility, queuing, and choice modeling theories to model consumers' behavior and to find solutions to the firm's profit maximization problem. *Managerial Implications*: The optimal strategy is to disclose either error-free delivery reliability information or no information at all. We also delineate conditions for each of the two strategies to dominate.

Keywords: service operations, delivery time, pricing, information disclosure, online retailing.

1 Introduction

In business models that involve waiting times, consumers are often sensitive both to promised delivery times and to deliveries being on time. For example, a survey found that 75% of U.S. online shoppers care about on-time delivery (Carollo 2018), hereafter referred to as *delivery reliability* (So 2000 and Marand, Li, and Thorstenson 2020). As noted by Cosgrove (2019), business firms can promise short delivery times to attract consumers. However, the probability of late delivery increases as promised delivery times are shortened, leading to a problematic trade-off.

A business firm may ease this trade-off by managing the delivery reliability information available to consumers. The firm can disclose true, error-free delivery reliability rates or may collect and publish error-prone, consumer-generated delivery-time ratings. Alternatively, the firm may choose to publish no delivery reliability information at all. These three strategies provide consumers with information sets that differ in terms of precision and thereby induce different consumer expectations.

Among the top ten online marketplaces by market share in the U.S. (according to eMarketer), two do not publish any ratings (Apple and Carvana), seven publish aggregate ratings that combine all aspects of the consumer experience (Amazon, Walmart, Target, Home Depot, Best Buy, Costco, and Kroger), and one publishes separate ratings for delivery performance (eBay). This variety of strategies suggests that a dominant strategy is unknown or it does not exist. Previous research has addressed separate parts of this problem by, for instance, assuming that delivery reliability information is exact (e.g., Ho and Zheng 2004), or that consumers are sensitive to delivery performance but not to promised delivery times (e.g., Ren, Huang, and Arifoglu 2018).

We propose and solve a theoretical model to find the optimal decisions of a firm that serves consumers who are sensitive to both promised delivery time and delivery reliability. The firm seeks to maximize its profits by setting the price, the promised delivery time, and the level of investment in the quality of the delivery reliability information. Accordingly, we specify the utility consumers expect to draw from buying the firm's offering, derive the demand function, and maximize the firm's profit to find optimal solutions. We verify the realism of our model by analyzing data from T-mall's retailing platform. In addition, we perform numerical studies to examine the firm's optimal decisions if the firm compensates consumers when it delivers after the promised delivery time.

Our analyses generate a number of managerial insights. We find that the firm's optimal decision is to make either full effort or zero effort to improve the quality of the delivery-time information. The decision depends directly on the cost of information quality, the dispersion of consumer valuations of the firm's offering, and consumer sensitivity to *delivery trust*, which we define as the delivery reliability expected by consumers. Profits decline with increasing costs of information quality but grow with increasing dispersion of consumer valuations and with increasing consumer sensitivity to delivery trust. We also show that the firm cannot change information quality to improve delivery-time performance and increase consumer demand simultaneously. Moreover, the firm may be worse off if consumer sensitivity to the promised delivery time increases. Finally, the presence of negative biases in consumers' delivery trust could make the no-effort decision more likely to be optimal.

These results contribute to a broad literature on how the amount of information available to consumers affects outcomes in queuing systems [see Ch. 3 of (Hassin 2016) for a review]. Existing studies consider different forms of information, such as the state of the system (Guo & Zipkin 2007, Economou & Kanta 2008), system parameters (Cui & Veeraraghavan 2016), and offering quality (Debo, Parlour & Rajan 2012, Guo, Haviv, Luo & Wang 2022). A first stream of research within this literature (e.g., So 2000 and Yu 2015) focuses on settings in which consumer decisions depend on promised delivery times and prices but not directly on delivery reliability. A second stream of work (e.g., Chatterjee, Slotnick, and Sobel 2002, Shang and Liu 2011) makes the opposite assumption that exact information on actual delivery reliability is available to consumers. A third stream of research (e.g., Huang and Chen 2015, Ren, Huang, and Arifoglu 2018) studies a middle ground in which consumers have partial knowledge of service quality. Studies in this stream, however, do not consider promised delivery time and how it affects the delivery reliability. Unlike extant publications, this work focuses on the availability of information on delivery reliability.

2 Model

We consider a queuing system in which a firm serves consumers by delivering its offerings (products or services) at service rate μ. The firm promises a delivery (sojourn) time t, sets a price p, and makes an investment ϕ in delivery reliability information quality. In practice, the firm can invest in improving the quality of delivery reliability information available to consumers by, for example, improving the accuracy of its consumer rating algorithms or by building infrastructure capable of reporting true delivery reliability (as T-mall did). Here, we abstract the level of investment as a decision variable $\phi \in [0, 1]$ without loss of generality, and define the three following scenarios: the *full-effort* scenario, $\phi = 1$; the *partial effort* scenario, $\phi \in (0, 1)$; and the *no-effort* scenario, $\phi = 0$.

The inter-arrival rate of consumers is $\lambda(p, t, \phi)$ (for short, λ). The promised delivery time t includes order processing and shipping times. Buyers pay only the product price as there are no shipping fees (as in T-mall's platform). The actual delivery time T often differs from the promised delivery time t and is naturally treated as a random variable (So 2000, Ho & Zheng 2004). The probability of delivering the offering within the promised delivery time, i.e., $q = \Pr(T \leq t)$, is the *delivery reliability*, which abstracts the fraction of consumers who are satisfied with the delivery time (Ho & Zheng 2004). We model the system as an $M/M/1$ queuing model. Hence,

$$q = 1 - \exp(-(\mu - \lambda)t). \tag{1}$$

Naturally, $t \geq 1/(\mu - \lambda)$, because, otherwise, $q < 1 - \exp(-1) = 63\%$ is very low. Our analysis of the data from a random sample of sellers at T-mall indicates that the mean and the standard deviation of the delivery reliability across sellers are 97.27% and 2.78%, respectively.

2.1 Information Quality and Inferred Delivery Reliability

We model consumer inferences of delivery reliability as a random variable R with consumer-specific realizations. Because R reflects a consumer's trust in the promised delivery time t, we refer to R as the "degree of delivery trust" (for similar uses of the term "trust," see, for example, Oliveira et al. 2017 and Ta, Esper, and Hofer 2018). Without loss of generality, R is a function of q and a random variable X that represents informational error. In general, R is increasing in q. An analysis of T-mall's data (see online Appendix A) confirms this logic and suggests that the relationship can be abstracted as linear. We accordingly define the random degree of delivery trust as

$$R \equiv \rho q + X = \rho[1 - \exp(-(\mu - \lambda)t)] + X, \tag{2}$$

where ρ is included to scale the effect of q on R. As discussed in online Appendix B, both $E(X)$ and $Var(X)$ are usually decreasing in information quality such that $E(X|\phi = 0) = \max_\phi E(X)$ and $Var(X|\phi = 0) = \max_\phi Var(X)$ when information quality is lowest; and $Var(X|\phi = 1) = 0$ and $E(X|\phi = 1)$ may be zero when information quality is highest.

We model the random variable X as $X = \delta_1 Z + \delta_2$, where δ_1 is a positive constant, δ_2 is a constant that can be positive, zero, or negative, and Z is a random variable that satisfies the following three conditions: (i) $\Pr(Z = 0|\phi = 1) = 1$; (ii) $E(Z|0 \leq \phi < 1) > 0$; and (iii) both $E(Z)$ and $Var(Z)$ are decreasing in ϕ. Because δ_2 could be negative, $E(X) = \delta_1 E(Z) + \delta_2$ may be positive, zero, or negative. Notably, when $\delta_1 = 1$ and $\delta_2 = 0$, $\phi = 1$ implies that $\Pr(X = 0) = 1$ and $R = \rho q$, which is the standard assumption in the literature (e.g., So 2000, Ho and Zheng 2004, Marand, Li, and Thorstenson 2020) and leads to closed forms for the demand function. The random variable Z satisfies conditions (i), (ii), and (iii) if it follows the $C(\phi)$ distribution (Cardell 1997).

Claim 1 *The $C(\phi)$ distribution has a single parameter $\phi \in [0,1]$ that determines both its central location and scale. If $Z \sim C(\phi)$, then $E(Z) = \gamma(1-\phi)$ and $Var(Z) = \pi^2(1-\phi^2)/6$, where $\gamma \equiv 0.57721$ is the Euler-Mascheroni constant. See online Appendix C for the derivation of these expressions. The mean and variance are both decreasing in ϕ.*

If $Z \sim C(\phi)$, the case $\phi = 1$ corresponds to the full-effort scenario, in which $E(X|\phi = 1) = \delta_2$ and $Var(X|\phi = 1) = 0$. In contrast, $\phi = 0$ represents the no-effort scenario, in which $C(0)$ is the standard Gumbel distribution and thus, $E(X|\phi = 0) = \delta_1\gamma + \delta_2$ and $Var(X|\phi = 0) = \delta_1^2\pi^2/6$. Values of $\phi \in (0,1)$ correspond to the partial-effort scenario. It follows that the parameter ϕ of the $C(\phi)$ distribution can assume the role of the firm's effort to improve information quality. We accordingly use the $C(\phi)$ distribution to model Z and use the distribution's parameter ϕ to represent the firm's investment in information quality.

2.2 The Consumer Utility Functions

Every arriving consumer considers both buying the firm's offering and balking (i.e., buying the outside good) and chooses the option that offers the highest expected utility.

2.2.1 Utility of Buying

The consumer's expected utility of buying the offering depends on p, t, and a base value α, i.e.,

$$U \equiv \alpha - p + \beta_r R - \beta_t t + Y = \alpha - p + \beta_r(\rho q + X) - \beta_t t + Y = V + \theta, \tag{3}$$

where $V \equiv \alpha - p + \beta_r(\rho q + \delta_2) - \beta_t t$ and $\theta \equiv \beta_r \delta_1 Z + Y$ are the deterministic and random components, respectively. Here, V is bounded from above by $\bar{V} \equiv \alpha - c + \beta_r(\rho + \delta_2) - \beta_t \underline{t}$, where $\underline{t} \geq 0$ denotes the shortest delivery time and c denotes the unit acquisition cost. The parameters $\beta_t \geq 0$ and $\beta_r \geq 0$ determine the importance of promised delivery time t and delivery trust R, respectively. The random variable Y represents consumer-specific valuations of the offering. As is standard (Ho & Zheng 2004, Jalili Marand, Li & Thorstenson 2020), we let $Y \sim Gumbel(\tau_Y, \omega_Y)$, where $\tau_Y \geq 0$ and $\omega_Y > 0$ are parameters denoting the location and scale of the distribution, respectively.

Let $\hat{\theta} \equiv \theta/(\beta_r\delta_1) = Z + \phi\hat{Y}$, for $0 < \phi < 1$, where $\hat{Y} \equiv Y/(\phi\beta_r\delta_1) \sim Gumbel$ with $E(\hat{Y}) = (\tau_Y + \gamma\omega_Y)/(\phi\beta_r\delta_1)$ and $Var(\hat{Y}) = [\omega_Y/(\phi\beta_r\delta_1)]^2\pi^2/6$. According to Cardell (1997), $\hat{\theta} \sim Gumbel$

with $Var(\hat{\theta}) = \omega_{\hat{\theta}}^2 \pi^2/6$, where $\omega_{\hat{\theta}}$ is the scale parameter. Using Claim 1, we obtain

$$\omega_{\hat{\theta}} = \frac{\sqrt{6(Var(Z) + \phi^2 Var(\hat{Y}))}}{\pi} = \frac{\sqrt{(\beta_r \delta_1)^2(1-\phi^2) + \omega_Y^2}}{\beta_r \delta_1},$$

and conclude that θ is Gumbel distributed with scale factor $\omega_\theta \equiv \sqrt{(\beta_r \delta_1)^2(1-\phi^2) + \omega_Y^2}$.

2.2.2 Utility of Balking

The utility of balking (buying nothing or a substitute offering) is $U_e \equiv V_e + Y_e$, where V_e and Y_e are the deterministic and random components, respectively. It is typically assumed that Y_e and θ are independently and identically distributed (i.e., $Y_e \sim Gumbel$ with scale ω_θ), which often holds in practice (see cases studied by Munizaga, Heydecker, and de Dios Ortuzar 2000, Severin, Louviere, and Finn 2001) and should hold for any well-specified utility function (see Train 2009, Ch. 3).

2.3 The Demand and Profit Functions

A consumer buys the firm's offering if $U - U_e = V - V_e + \tilde{\theta} \geq 0$ where $\tilde{\theta} \equiv \theta - Y_e$. Otherwise, the consumer balks. Because θ and Y_e are i.i.d. Gumbel variables, we have that $\tilde{\theta} \sim Logistic(0, \omega_\theta)$ and as a result (Train 2009) the probability of the "buy" alternative is

$$S \equiv \Pr\left(\frac{\tilde{\theta}}{\omega_\theta} \geq \frac{V_e - V}{\omega_\theta}\right) = \frac{1}{1 + \exp[-(V - V_e)/\omega_\theta]}.$$

Letting Λ denote the market size, we find

$$\lambda(p, t, \phi) = \Lambda S \leq \bar{S}\Lambda, \tag{4}$$

where $\bar{S} \equiv 1/(1 + \exp[-(\bar{V} - V_e)/\omega_\theta])$ denotes the firm's maximum market share. Hence, the firm's service rate μ satisfies $\mu \geq \bar{S}\Lambda$. The firm's profit is $\Pi(p, t, \phi) = \lambda(p, t, \phi)(p - b\phi - c)$, where $b > 0$ represents the cost per unit of such effort or, for short, the unit effort cost.

3 Main Analyses

We first condition demand on ϕ and solve $\max_{p,t} \Pi(p, t) \equiv \lambda(p, t|\phi)(p - b\phi - c)$ to find $p^*(\phi)$ and $t^*(\phi)$. Then, we solve $\max_\phi \Pi(p^*(\phi), t^*(\phi), \phi)$ to obtain ϕ^* and compute $p^* = p^*(\phi^*)$ and $t^* = t^*(\phi^*)$.

3.1 Step 1: Derivation of $p^*(\phi)$ and $t^*(\phi)$

Lemma 1 *We can determine unique $q^*(\phi)$ and $p^*(\phi)$ as follows.*

1. *If $\mu < \bar{S}\Lambda + \beta_t/(\beta_r\rho)$, then $t^*(\phi) = \underline{t}$ and $p^*(\phi)$ is given by the unique solution to:*

$$p = b\phi + c + \underline{t}\beta_r\rho\lambda(p,\underline{t})\exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \frac{\omega_\theta\Lambda}{\Lambda - \lambda(p,\underline{t})}. \tag{5}$$

2. *If $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$, then one first solves the following equation to find $t^*(p,\phi)$:*

$$\beta_r\rho(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) = \beta_t. \tag{6}$$

Then one can obtain the unique price function $p^*(\phi)$ by solving the following equation for p:

$$p = b\phi + c + \frac{\beta_t\lambda(p)t^*(p)}{\mu - \lambda(p)} + \frac{\omega_\theta\Lambda}{\Lambda - \lambda(p)}. \tag{7}$$

Finally, $t^*(\phi) = t^*(p^*(\phi),\phi)$.

The solution when $\mu < \bar{S}\Lambda + \beta_t/(\beta_r\rho)$ is trivial and unrealistic in most business contexts of interest because it implies that the firm can deliver its offerings instantaneously. Accordingly, subsequent analyses focus on the scenario $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$. Using (6), we find that $1/(\mu - \lambda^*(\phi)) \leq \beta_r\rho\exp(-1)/\beta_t$, i.e., the upper bound of the average delivery time is $\beta_r\rho\exp(-1)/\beta_t$. Because $\max_{x\in\mathbb{R}}(x\exp(-x)) = \exp(-1)$, we have $\beta_t t^*(\phi)/(\beta_r\rho) \leq \exp(-1)$ or $t^*(\phi) \leq \beta_r\rho\exp(-1)/\beta_t$.

3.2 Step 2: Derivation of ϕ^*, $p^*(\phi^*)$, and $t^*(\phi^*)$

3.2.1 The Impact of ϕ

The direct derivation of ϕ^*, $p^*(\phi^*)$, and $t^*(\phi^*)$ is not tractable. We instead characterize these functions by studying the impact of ω_θ and leveraging the result that $\partial\omega_\theta/\partial\phi = -(\beta_r\delta_1)^2\phi/\omega_\theta < 0$, i.e., ω_θ is monotonically decreasing in ϕ over the range $\phi \in (0,1)$. Using (1) and (6), we write $q^*(\phi) = 1 - \beta_t/\left[\beta_r\rho(\mu - \lambda^*(\phi))\right]$ and differentiate it once w.r.t. ω_θ to obtain

$$\frac{\partial q^*(\phi)}{\partial\omega_\theta} = -\frac{\beta_t}{\beta_r\rho(\mu - \lambda^*(\phi))^2}\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}, \tag{8}$$

which implies that the impact of ω_θ on $q^*(\phi)$ and the impact of ω_θ on $\lambda^*(\phi)$ have opposite signs. Hence, the impact of ϕ on $q^*(\phi)$ and that of ϕ on $\lambda^*(\phi)$ differ likewise.

Lemma 2 *Consider the inequalities:*

$$\frac{b\omega_\theta^2}{(\beta_r\delta_1)^2\phi} \geq V - V_e, \tag{9}$$

$$\frac{\partial\lambda^*}{\partial\omega_\theta} > \frac{\rho\gamma\omega_\theta(\mu - \lambda^*(\phi))^2}{\phi\beta_r\beta_t\delta_1}. \tag{10}$$

If ϕ is set exogenously, then we have the following results.

1. *If condition (9) holds, then $t^*(\phi)$, $\lambda^*(\phi)$, and $\Pi^*(\phi)$ decrease with ϕ; and $q^*(\phi)$ increases with ϕ. In addition, if (10) holds, then $E(R^*)$ increases with ϕ; otherwise, $E(R^*)$ decreases with ϕ.*
2. *If condition (9) does not hold, then $p^*(\phi)$, $t^*(\phi)$, $\lambda^*(\phi)$, and $\Pi^*(\phi)$ increase with ϕ; but, $q^*(\phi)$ and $E(R^*)$ decrease with ϕ. We also find that V is decreasing in ϕ but $V > V_e$.*

Lemma 2 thus defines two scenarios. If $V \leq V_e$, then (9) holds because $b\omega_\theta/[(\beta_r\delta_1)^2\phi] \geq 0$. If instead $V > V_e$, we use $\partial\omega_\theta/\partial\phi = -(\beta_r\delta_1)^2\phi/\omega_\theta$ to find

$$\kappa_1 \equiv -\frac{\partial\omega_\theta/\omega_\theta}{\partial\phi/\phi} \leq \kappa_2 \equiv \frac{b\phi}{V - V_e}, \tag{11}$$

where κ_1 denotes the elasticity of ω_θ with respect to ϕ, and κ_2 denotes the firm's information quality effort relative to consumers' information-independent, net utility gain.

We learn from (11) that the firm's information quality effort elasticity of consumers' reliability inference is no larger than the firm's effort relative to consumers' information-independent net gain. When (9) holds, the firm's information quality efforts are inefficient ($\kappa_1/\kappa_2 \leq 1$). To attract consumers, the firm reduces the promised delivery time, which, nonetheless, reduces demand and profits. When (9) does not hold, the firm's efforts are efficient ($\kappa_1/\kappa_2 > 1$) and the firm should increase the promised delivery time to mitigate the reduced reliability, in agreement with (1).

3.2.2 The Optimal Decisions

Theorem 1 *If*

$$b\omega_Y^2/(\beta_r\delta_1)^2 \geq V_1 - V_e, \tag{12}$$

where $V_1 \equiv V|_{\phi=1}$, then $\phi^* = 0$; otherwise, $\phi^* = \arg\max_{\phi\in\{0,1\}} \Pi^*(\phi)$.

Theorem 1 indicates that the firm should make a full effort to inform consumers of the true delivery reliability, or make zero effort. It is suboptimal for the firm to make a partial effort. The intuition behind this result is as follows. Condition (12) holds when: (a) information quality is sufficiently costly (i.e., b should be sufficiently high); (b) consumers care little enough about reliability (i.e., $\beta_r\delta_1$ should be sufficiently small); and (c) consumer individual valuations of the firm's offerings are sufficiently dispersed (i.e., ω_Y should be sufficiently large); or, alternatively, when (d) $V_1 \leq V_e$. If either all of (a)-(c) or (d) hold, then, for any value of ϕ, the firm can always increase its profit by lowering the quality of information, thus ending up making no investment in information quality. Otherwise, when neither (a)-(c) or (d) are satisfied, Condition (12) does not hold and the firm profits from making high-quality information available to consumers.

We define $p^* = p^*(\phi^*)$, $t^* = t^*(\phi^*)$, $\lambda^* = \lambda^*(\phi^*)$, $\Pi^* = \Pi^*(\phi^*)$, and $q^* = q^*(\phi^*) = 1 - \exp(-(\mu - \lambda^*)t^*) = 1 - \beta_t/[\beta_r\rho(\mu - \lambda^*)]$; thus, the deterministic component of the degree of delivery trust R is

$$\rho q^* = \rho - (\beta_t/\beta_r)(\mu - \lambda^*)^{-1}, \tag{13}$$

and declines linearly on the average delivery time $(\mu - \lambda^*)^{-1}$ by a factor equal to the ratio β_t/β_r. That is, the average delivery time has a large impact on the degree of delivery trust if the promised delivery time is more important to consumers than the degree of delivery trust.

Moreover, if $\lambda_1^* \equiv \lambda^*|_{\phi^*=1} \leq \lambda^*|_{\phi^*=0} \equiv \lambda_0^*$, then $q_1^* \equiv q^*|_{\phi^*=1} \geq q^*|_{\phi^*=0} \equiv q_0^*$. That is, $(q_0^* - q_1^*)(\lambda_0^* - \lambda_1^*) \leq 0$, i.e., the firm cannot simultaneously increase both delivery-time performance and demand rate by choosing between $\phi^* = 0$ and $\phi^* = 1$. Likewise, the number of late orders is $(1 - q_i^*)\lambda_i^* = \beta_t/(\beta_r\rho) \times \lambda_i^*/(\mu - \lambda_i^*)$, for $i = 0, 1$, where $\lambda_i^*/(\mu - \lambda_i^*)$ is the expected delivery volume. Hence, the larger the delivery volume, the fewer the consumers who receive their orders on time. The firm faces a trade-off between demand volume and delivery performance.

4 Sensitivity Analysis

We define the β_i-elasticity ($i = t, r$) of the optimal promised delivery time t^* and the β_i-elasticity of demand λ^*, respectively, as $\eta_{t^*,\beta_i} = (\partial t^*/t^*)/(\partial\beta_i/\beta_i)$ and $\eta_{\lambda^*,\beta_i} = (\partial\lambda^*/\lambda^*)(\partial\beta_i/\beta_i)$, for $i = t, r$.

Corollary 1 *If the value of β_t increases: (i) the promised delivery time t^* shortens and the demand λ^* decreases; (ii) the firm's optimal price p^* and profits Π^* decrease if $\eta_{t^*,\beta_t} \leq -1$ but may increase otherwise; and (iii) q^* increases if $\eta_{\lambda^*,\beta_t} \geq (\lambda^* - \mu)/\lambda^*$ but decreases otherwise.*

Corollary 1 indicates that, naturally, as consumer utility becomes more sensitive to the promised delivery time, the firm shortens it. However, the demand also decreases, mainly because a shorter promised delivery time may translate into a lower reliability. The optimal price could increase but could also decrease, when the optimal promised delivery time is *elastic* with respect to consumer sensitivity to the promised delivery time. Because $(\mu - \lambda^*)/\lambda^* = (1 - \lambda^*/\mu)/(\lambda^*/\mu)$, where $1 - \lambda^*/\mu$

and λ^*/μ represent the probabilities for the idle and busy statuses respectively, we can interpret $(\mu - \lambda^*)/\lambda^*$ as the "odds of service idleness." In practice, the odds of service idleness are likely to be small and therefore, if $-\eta_{\lambda^*,\beta_t} \leq (\mu - \lambda^*)/\lambda^*$, then consumer demand is likely to be inelastic to the promised delivery time, and delivery reliability q^* is likely to increase with β_t.

Corollary 2 *When β_r increases, we find that*

1. *for both $\phi^* = 0$ and $\phi^* = 1$, if*

$$q^* \leq \frac{\beta_t \lambda^* - \beta_r \delta_2 (\mu - \lambda^*)^2}{\beta_t \mu - \beta_r \delta_2 (\mu - \lambda^*)^2}, \tag{14}$$

   then q^* increases, λ^* decreases, and p^*, t^*, and Π^* may increase; and

2. *for $\phi^* = 1$, if the inequality (14) does not hold and $1/(\mu - \lambda_1^*)$ is sufficiently small, then $p_1^* \equiv p^*|_{\phi^*=1}$, $t_1^* \equiv t^*|_{\phi^*=1}$, λ_1^*, and Π_1^* increase. In addition, q_1^* increases only when $\eta_{\lambda_1^*,\beta_r} \leq (\mu - \lambda_1^*)/\lambda_1^*$.*

We note that the right-hand side of (14) is smaller than the service utilization λ^*/μ. Thus, Corollary 2 indicates that, if service utilization is no smaller than delivery reliability q^*, increased consumer sensitivity to reliability may induce the firm to improve it. However, demand may decline because of higher prices and longer promised delivery times. Yet, profit may increase. If service utilization is smaller than reliability, then the results could differ for $\phi = 0$ and $\phi = 1$.

Corollary 3 *When μ increases, for both $\phi^* = 0$ and $\phi^* = 1$: (i) t^* decreases and λ^* increases by less than the increase in μ, i.e., $0 < \partial\lambda^*/\partial\mu <$; (ii) q_i^* $(i = 0, 1)$ increases; and (iii) p^* and Π^* may increase if the expected delivery time $1/(\mu - \lambda_i^*)$ is sufficiently short but may decrease otherwise.*

Because $0 < \partial\lambda_i^*/\partial\mu < 1$ $(i = 0, 1)$, we have that $\partial[1/(\mu-\lambda_i^*)]/\partial\mu = -[1/(\mu-\lambda_i^*)^2]\,(1 - \partial\lambda_i^*/\partial\mu) < 0$, which means that when μ is sufficiently large, $1/(\mu - \lambda_i^*)$ is sufficiently small and thus the firm's optimal price and expected profit are both increasing in μ.

Corollary 4 *When ω_Y increases, for both $\phi^* = 0$ and $\phi^* = 1$: (i) both t^* and p^* increase; (ii) q^* decreases; and (iii) λ^* and Π^* increase.*

Corollary 4 indicates that increased dispersion in consumer valuations of the firm's offerings leads to higher demand because it implies that a larger number of consumers predict utility levels that are far above the utility of the outside good. A higher predicted utility allows the firm to charge higher prices and offer longer delivery times without offsetting gains in demand. Yet, longer promised delivery times are not sufficient to mitigate the increased demand and delivery reliability drops. The firm thus capitalizes on higher prices and increased demand to reap higher profits.

5 Numerical Studies

We numerically investigate the sensitivity of our results to model parameters and, in particular, to the possibility that the firm compensates consumers by reimbursing a fixed amount when a delivery is late. Letting d denote the penalty for late delivery, we compute the expected cost of late deliveries as $\Pr(T > t) \times d = d(1-q) = d\exp(-(\mu - \lambda(p,t,\phi))t)$. Likewise, the consumer utility function is $\hat{U} = \alpha - p + d(1-q) + \beta_r \rho q - \beta_t t + \beta_r X + Y = \hat{V} + \theta$, where $\hat{V} \equiv \alpha + d - p + (\beta_r\rho - d)q - \beta_t t$ and $\theta = \beta_r X + Y$ as defined in Section 2.2.1. The firm's optimization problem is

$$\max_{p,t,\phi} \hat{\Pi}(p,t,\phi) \equiv \lambda(p,t,\phi)[p - b\phi - d(1-q) - c]. \tag{15}$$

We set $\delta_1 = 1$ and $\delta_2 = 0$ (i.e., $X = Z$), $[\Lambda, \alpha, \beta_r, \beta_t, \rho, V_e, \omega_Y, \mu, c, d] = [25, 5, 5, 5, 5, 5, 5, 100, 5, 5]$ to define our baseline scenario. For sake of consistency with our analytical results, we focus on parameters d, β_r, β_t, μ, and ω_Y. Similar results for other parameters and for alternative distributions of Z appear in online Appendix E. Starting with the baseline scenario, we vary the values of each focal parameter over the set $\{0, 5, 10\}$. For each combination of parameter values, we compute and plot $\hat{\Pi}^*$ over the range $\phi \in [0,1]$. We numerically solve (4) and (15) using the logit formula for the $C(\phi)$ distribution and simulation for other distributions. We extend the algorithm proposed by Ho and Zheng (2004) as follows.

Algorithm 1 *In Step 0, we set* $p^n = 1$ *and* $t^n = 1$*, for* $n = 0$*. In Step 1, we compute* $\lambda^n(p^n, t^n)$*. In Step 2, we find* $\{p^{n+1}, t^{n+1}\} = \arg\max_{p,t} \hat{\Pi}(p,t,\lambda^n)$*. In Step 3, if* $p^{n+1} = p^n$ *and* $t^{n+1} = t^n$*, then* $p^* = p^n$*,* $t^* = t^n$*, and we stop; otherwise, we let* $n = n+1$ *and go to Step 1.*

Numerical results appear in Figure 1 and show that, in every scenario, $\hat{\Pi}^*$ is convex for $\phi \in (0,1)$. This holds true for all values of d and other parameters. Furthermore, the effects of changes to parameters β_r, β_t, μ, and ω_Y are as predicted by our analytical results.

We report three more sets of numerical studies to illustrate possible shapes of the profit function and to assess the effect of the sign of $E(X)$. The three sets of scenarios correspond to $b \in \{0, 1, 5\}$. We keep $\delta_1 = 1$ but let $\delta_2 \in \{-1, -0.5, 0, 0.5, 1\}$ to allow for $E(X) < 0$. As Figure 2 illustrates, $\delta_2 \geq 0$ implies that the optimal information quality level is either $\phi^* = 1$ (when $b = 0, 1$) or $\phi^* = 0$ (when $b = 5$). Results for $\delta_2 < 0$ are similar, except that $\phi^* = 0$ when $b = 1$. This suggests that

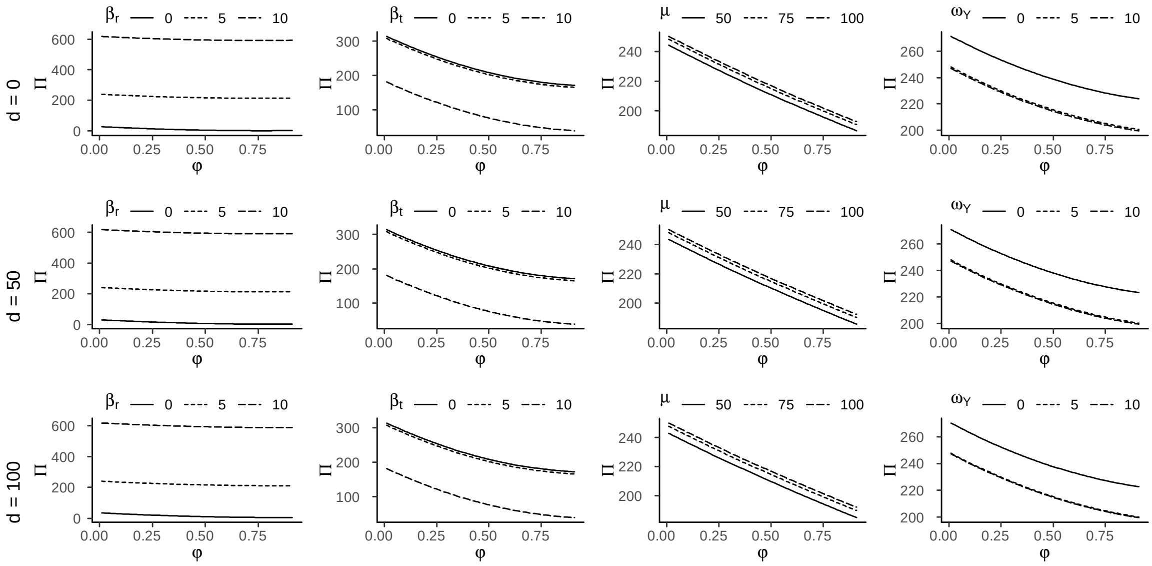

Figure 1: The response of optimal profits to information quality for different parameter values.

negative biases in consumer inferences of delivery reliability may make the no-effort strategy more likely to be optimal.

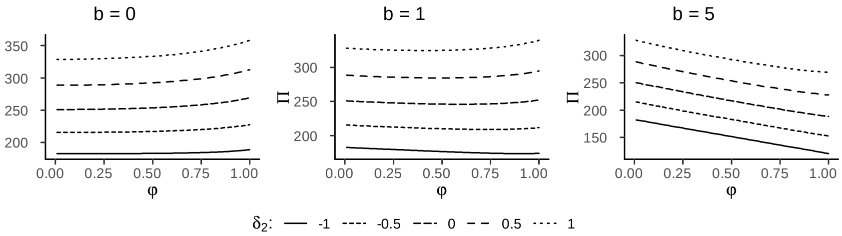

Figure 2: Possible shapes for the firm's profit function $\Pi^*(\phi)$ and the effect of the sign of δ_2.

6 Summary and Concluding Remarks

We explore the optimal pricing, promised delivery time, and delivery information quality effort decisions of a firm that serves consumers sensitive to price, promised delivery time, and delivery reliability. We contribute to the literature with both analytic modelling innovations and managerial insights. Specifically, our proposed model abstracts the different degrees of information quality observed in practice by regarding the error in delivery reliability information as a continuous random variable. The distribution of this random variable depends on a continuous measure of the firm's information quality effort. To incorporate this additional source of uncertainty into the standard random utility model, we capitalize on the special properties of the $C(\phi)$ distribution (Cardell 1997). This methodological innovation allows us to solve the model analytically and characterize the optimal decisions as well as the optimal reliability and profit. To the best of our knowledge, this work is the first to incorporate the $C(\phi)$ distribution into analytic models in the operations management field.

We obtain a number of managerial insights that can be helpful to managers of digital businesses. The five main insights are summarized as follows. First, our results imply that it is optimal for an online merchant to spend either full effort or no effort to improve the precision of consumer perceptions of delivery reliability. That is, the firm should not consider a partial effort. Whether it is optimal to make the full effort or no effort depends on many factors but mainly on the cost of information quality and the dispersion of consumer valuations. The higher these two main factors are, the less likely the firm should make a full effort. This suggests that the estimation of cost and consumer characteristics should receive especial attention from managers.

Second, the factors that influence the firm's profit the most are consumer sensitivity to delivery trust and the correlation between consumer inferences of delivery reliability and actual delivery reliability (see online Appendix E). Profits increase with growing sensitivity to delivery trust and with a strengthening correlation between inferred and actual delivery reliability. Yet, negative biases in consumer inferences of delivery reliability harm profits and make the no-effort decision more likely to be optimal.

Third, the firm cannot simultaneously increase both delivery reliability and consumer demand by changing the information quality level. This interesting result indicates that the consumer who is more likely to trust the delivery-time information is not more likely to buy, because the consumer's purchase also depends on other factors such as price, promised delivery time, etc. Hence, the results suggest that online merchants may need to determine whether profits or market share are more important before deciding on their delivery-time information efforts.

Fourth, we find that a higher service rate could help simultaneously raise delivery reliability, consumer demand, as well as the firm's profit. This finding emphasizes the importance of investing in service capacity. Fifth, high heterogeneity in consumer valuations of the firm's offerings harms profits. This suggest that, ceteris paribus, online merchants should favor markets in which consumer valuations of their products are as uniform as possible.

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References

Cardell, N. S. (1997). Variance components structures for the extreme-value and logistic distributions with application to models of heterogeneity, *Econometric Theory* **13**(2): 185–213.

Carollo, R. (2018). Consumers want faster delivery and they're willing to pay for it, Dropoff, Inc. https://www.dropoff.com/blog/retail-delivery-consumer-survey-shoptalk-2018 (URL last accessed on June 25, 2024).

Chatterjee, S., Slotnick, S. A. & Sobel, M. J. (2002). Delivery guarantees and the interdependence of marketing and operations, *Production and Operations Management* **11**(3): 393–410.

Cosgrove, E. (2019). Deliv CEO shares 4 reasons last-mile delivery costs skyrocket. https://www.supplychaindive.com/news/deliv-last-mile-delivery-costs-skyrocket/562969/ (URL last accessed on June 25, 2024).

Cui, S. & Veeraraghavan, S. (2016). Blind queues: The impact of consumer beliefs on revenues and congestion, *Management Science* **62**(12): 3656–3672.

Debo, L. G., Parlour, C. & Rajan, U. (2012). Signaling quality via queues, *Management Science* **58**(5): 876–891.

Economou, A. & Kanta, S. (2008). Optimal balking strategies and pricing for the single server markovian queue with compartmented waiting space, *Queueing Systems* **59**(3): 237–269.

Guo, P., Haviv, M., Luo, Z. & Wang, Y. (2022). Optimal queue length information disclosure when service quality is uncertain, *Production and Operations Management* **31**(5): 1912–1927.

Guo, P. & Zipkin, P. (2007). Analysis and comparison of queues with different levels of delay information, *Management Science* **53**(6): 962–970.

Hassin, R. (2016). *Rational Queueing*, 1st edn, CRC press, New York.

Ho, T. H. & Zheng, Y. (2004). Setting customer expectation in service delivery: An integrated marketing-operations perspective, *Management Science* **50**(4): 479–488.

Huang, T. & Chen, Y.-J. (2015). Service systems with experience-based anecdotal reasoning customers, *Production and Operations Management* **24**(5): 778–790.

Jalili Marand, A., Li, H. & Thorstenson, A. (2020). Competing on price, speed, and reliability: How does bounded rationality matter?, *Journal of the Operational Research Society* pp. 1–14.

Munizaga, M. A., Heydecker, B. G. & de Dios Ortúzar, J. (2000). Representation of heteroskedasticity in discrete choice models, *Transportation Research Part B* **34**: 219–240.

Oliveira, T., Alhinho, M., Rita, P. & Dhillon, G. (2017). Modelling and testing consumer trust dimensions in e-commerce, *Computers in Human Behavior* **71**: 153–164.

Ren, H., Huang, T. & Arifoglu, K. (2018). Managing service systems with unknown quality and customer anecdotal reasoning, *Production and Operations Management* **27**(6): 1038–1051.

Severin, V., Louviere, J. & Finn, A. (2001). The stability of retail shopping choices over time and across countries, *Journal of Retailing* **77**: 185–202.

Shang, W. & Liu, L. (2011). Promised delivery time and capacity games in time-based competition, *Management Science* **57**(3): 599–610.

So, K. C. (2000). Price and time competition for service delivery, *Manufacturing & Service Operations Management* **2**(4): 392–409.

Ta, H., Esper, T. L. & Hofer, A. R. (2018). Designing crowdsourced delivery systems: The effect of driver disclosureand ethnic similarity, *Journal of Operations Management* **60**: 19–33.

Train, K. (2009). *Discrete Choice Methods with Simulation*, 2nd edn, Cambridge University Press, Cambridge.

Yu, J. (2015). *Delivery service assortment and product pricing in online retailing: The impact of pricing flexibility and customer rating*, Master's thesis, The University of British Columbia.

Online Appendices

"Disclosing Delivery Performance Information when Consumers are Sensitive to Promised Delivery Time, Delivery Reliability, and Price"

Appendix A Average Delivery-Time Ratings and Conformance

We used T-mall/Cainiao data to explore the relationship between average delivery-time ratings and delivery reliability. First, we compute the reliability of each merchant in the data set and the average of the individual-level ratings provided by buyers, across all orders with complete data. This yields 3,665 observations. Then we perform an OLS regression of average delivery-time ratings on reliability. The estimated regression equation is

$$\text{average delivery-time rating} = \underset{(0.060)}{4.396} + \underset{(0.062)}{0.438} \times \text{reliability} + \epsilon,$$

where the standard errors of the estimates appear in parenthesis. The coefficient of reliability is highly significant (p-value$<$ 0.0001), indicating that there is a positive and statistically significant relationship between reliability and ratings. The intercept is positive, large, and statistically significant, so that ratings are positive even when reliability is zero. This implies that ratings exhibit a significant upward bias. We then inspect the residuals to verify that the conditions for inference are satisfied (residuals should be normally distributed, homoskedastic, independent, and the relationship linear). We note, however, that the distribution of reliability is highly skewed, with 50% of the observations having a value above 0.98 and fewer than 5% of the observations having values below 0.90. This implies that residuals could appear heteroskedastic simply because dispersion is normally proportional to sample size and therefore we could expect higher dispersion for high levels of reliability. For this reason, we randomly sample 500 observations from the data with probability weights inversely proportional to the square of their reliability values. We then replicate the regression analysis and obtain the estimated regression equation

$$\text{average delivery-time rating} = \underset{(0.113)}{4.248} + \underset{(0.116)}{0.593} \times \text{reliability} + \epsilon.$$

Once again, we find evidence of a positive and statistically-significant correlation (p-value$<$ 0.0001) between reliability and average delivery-time ratings. The standardized residuals do not exhibit any clear sign of homoskedasticity, non-linearities, or non-normality. Data are not ordered and the possibility of autocorrelation is ruled out. The conditions for inference are satisfied.

Appendix B Information Quality and Inference Distribution

The propositions that $E(X)$ and $Var(X)$ are both positive and decreasing on information quality are backed by several studies in the literature. For example, field experiments in the context of consumer expectations of prices show that making high-quality information available to consumers reduces the heterogeneity and bias of their expectations (Fuster et al. 2002). In the context of online retailing, consumer expectations exhibit a positive bias because online reviews influence consumer expectations and online reviews are biased upward (see Park et al. 2018 and references therein).

Similarly, research has shown that both the mean and dispersion of consumer attitudes towards offerings decline with the quality of reviews. This occurs because high-quality negative reviews affect attitude (expectations) more than low-quality reviews do, whereas positive reviews are less effective regardless of their quality (Lee et al. 2008). Because attitudes informed by ratings are normally biased upwards, the negative effect of increased information quality on attitudes should reduce such positive bias. Conversely, lowering the quality of online reviews can increase the bias and variance of subsequent evaluations (Muchnik et al. 2013). In summary, the literature indicates that expectations exhibit a positive bias, which decreases as information quality increases. The dispersion (heterogeneity) of these expectations also decreases as information quality increases.

One may note that the firm may benefit from increasing information quality even if the optimal degree of delivery trust decreases when both $E(X)$ and $Var(X)$ decrease. Specifically, information error inflating the degree of trust can harm the firm because it implies that consumers "overestimate" delivery reliability q. When consumers overestimate the reliability, they are less satisfied with the actual delivery service and provide lower delivery ratings. It follows that a higher degree of trust resulting from a larger informational error makes the firm worse off. That is, a "lower degree of trust" is better for the firm than an "inflated degree of trust." Therefore, the firm has an incentive to improve information quality (i.e., increase the value of ϕ), reducing $E(X)$ and $Var(X)$ and deflating the degree of trust. Nonetheless, the firm can benefit from a high degree of trust that is made by a large delivery reliability q.

References
[1] Fuster, A., Perez-Truglia, R., Wiederholt, M. and Zafar, B., 2022. Expectations with endogenous information acquisition: An experimental investigation. *Review of Economics and Statistics*, 104(5), pp.1059-1078.
[2] Lee, J., Park, D. H. and Han, I., 2008. The effect of negative online consumer reviews on product attitude: An information processing view. *Electronic Commerce Research and Applications*, 7(3), pp.341-352.
[3] Muchnik, L., Aral, S. and Taylor, S. J., 2013. Social influence bias: A randomized experiment. *Science*, 341(6146), pp.647-651.
[4] Park, K., Cha, M. and Rhim, E., 2018, April. Positivity bias in customer satisfaction ratings. In Companion Proceedings of the The Web Conference 2018 (pp. 631-638).

Appendix C The Mean and Variance of the $C(\phi)$ Distribution

Cardell (1997) derived the characteristic function of the $C(\phi)$ distribution as

$$\psi_\phi(\tau) = \Gamma(1 - \iota\tau)/\Gamma(1 - \iota\phi\tau), \tag{16}$$

where $\iota \equiv \sqrt{-1}$ is the imaginary unit, $\Gamma(z) \equiv \int_0^\infty x^{z-1} \exp(-x)dx$ is the standard gamma function, and $\tau \in R$ is the argument of the characteristic function. However, Cardell (1997) did not derive closed-form expressions for the mean and variance and these are not readily available in the literature. (Cardell did show that, although there is no closed-form cumulative distribution function for $C(\phi)$, there is a closed-form probability density function.)

Next, we derive closed-form expressions of the mean and variance of Z. From (16), the first and second moments of the $C(\phi)$ distribution are given by

$$\begin{cases} E(Z) & = \iota^{-1} \times \left.\dfrac{\partial \psi_\phi(\tau)}{\partial \tau}\right|_{\tau=0} & = \gamma(1-\phi), \\ E(Z^2) & = \iota^{-2} \times \left.\dfrac{\partial^2 \psi_\phi(\tau)}{\partial \tau^2}\right|_{\tau=0} & = \dfrac{(1-\phi)[\pi^2(1+\phi)+6\gamma^2(1-\phi)]}{6}, \end{cases}$$

where $\gamma \equiv 0.57721$ is the Euler-Mascheroni constant. The mean of Z is given by the first moment $E(Z)$ and, letting σ_Z denote the standard deviation of Z, we have the variance as

$$Var(Z) = \sigma_Z^2 = \pi^2(1-\phi^2)/6. \tag{17}$$

It follows that $\lim_{\phi\to 1} E(Z) = 0$ and $\lim_{\phi\to 1} Var(Z) = 0$, which is consistent with Cardell's finding, i.e., $\phi = 1$ implies that $\Pr(Z = 0) = 1$. In addition, we find that $\lim_{\phi\to 0} E(Z) = \gamma$ and $\lim_{\phi\to 0} Var(Z) = \pi^2/6$. This also is consistent with Cardell's finding: the $C(\phi = 0)$ distribution is the standard Gumbel distribution.

Appendix D Proofs

Proof of Lemma 1. Given the value of p, we compute the first and second-order derivatives of $\Pi(p,t)$ w.r.t. t as

$$\frac{\partial \Pi(p,t)}{\partial t} = \frac{\partial \lambda(p,t)}{\partial t}(p - b\phi - c) \text{ and } \frac{\partial^2 \Pi(p,t)}{\partial t^2} = \frac{\partial^2 \lambda(p,t)}{\partial t^2}(p - b\phi - c),$$

where $\lambda(p,t) \equiv \lambda(p,t|\phi)$. It follows from expression (4) that $\lambda(p,t,\phi) = \Lambda/[1+\exp(-(V-V_e)/\omega_\theta)]$, from which we obtain $V = V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$. For simplicity, we hereafter define $K \equiv V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$. We differentiate K once w.r.t. t to obtain

$$\frac{\partial K}{\partial t} \equiv \frac{\omega_\theta \Lambda}{\lambda(p,t)(\Lambda - \lambda(p,t))} \frac{\partial \lambda(p,t)}{\partial t}.$$

Note that $V = \alpha - p + \beta_r(\rho q + \delta_2) - \beta_t t$, where $q = 1 - \exp(-(\mu-\lambda)t)$ as given in (1). We then differentiate V once w.r.t. t to obtain

$$\frac{\partial V}{\partial t} = \beta_r \rho \exp(-(\mu - \lambda(p,t))t)\left[\mu - \lambda(p,t) - \frac{\partial \lambda(p,t)}{\partial t} t\right] - \beta_t.$$

Solving $\partial V/\partial t = \partial K/\partial t$ for $\partial \lambda(p,t)/\partial t$, we obtain

$$\frac{\partial \lambda(p,t)}{\partial t} = \frac{\lambda(p,t)(\Lambda - \lambda(p,t))\xi}{\beta_r \rho t \lambda(p,t)(\Lambda - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) + \omega_\theta \Lambda},$$

where $\xi \equiv \beta_r \rho(\mu - \lambda(p,t))\exp(-(\mu-\lambda(p,t))t) - \beta_t$. One can easily note that the sign of $\partial\lambda(p,t)/\partial t$ only depends on the sign of ξ. Because $S\Lambda \le \bar{S}\Lambda$, we have $\mu - \lambda(p,t) \ge \mu - \bar{S}\Lambda$. Noting that, for $a > 0$ and $ax \ge 1$, the function $x\exp(-ax)$ is decreasing in x, we find that, because $t \ge 1/(\mu - \lambda(p,t))$, then $(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) \le (\mu - \bar{S}\Lambda)\exp(-(\mu - \bar{S}\Lambda)t)$. In addition, because $\exp(-(\mu - \bar{S}\Lambda)t) \leq 1$, we have $(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) \leq (\mu - \bar{S}\Lambda)$. Multiplying both side of the inequality by $\beta_r \rho$, we obtain

$$\xi \leq \beta_r \rho(\mu - \bar{S}\Lambda) - \beta_t. \tag{18}$$

Thus, for a given value of p, if $\beta_r \rho(\mu - \bar{S}\Lambda) - \beta_t < 0$, or $\mu < \bar{S}\Lambda + \beta_t/(\beta_r \rho)$, then $\xi < 0$ or $\partial\lambda(p,t)/\partial t < 0$; and t should be $\underline{t}$, i.e., $t^*(p) = \underline{t}$.

Otherwise, if $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$, then the RHS of the inequality in (18) is non-negative and therefore ξ is no larger than a non-negative number. It remains to show that there exists at least one solution for $\xi = 0$. As argued above, $(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t)$ is decreasing in t; that is, ξ decreases with t. Because $0 \leq \lambda(p,t) \leq \bar{S}\Lambda$,

$$\lim_{t\to+\infty} \xi = \lim_{t\to+\infty} [\beta_r\rho(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) - \beta_t] = -\beta_t < 0.$$

Noting that $t \in [\underline{t}, +\infty)$ and $\underline{t} \geq 0$, we should also examine if $\xi|_{t\to 0} \geq 0$. Since $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$ and $0 \leq \lambda(p,t) \leq \bar{S}\Lambda$, $\mu - \lambda(p,t) \geq \beta_t/(\beta_r\rho)$. In addition, when $t \to 0$, $\exp(-(\mu - \lambda(p,t))t) \to 1$ and thus $(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) \geq \beta_t/(\beta_r\rho)$, which means $\xi|_{t\to 0} \geq 0$.

These results imply that we can find at least one solution satisfying $\xi = 0$. Hence, there is at least one solution for the first order condition $d\lambda(p,t)/dt = 0$. For any such solution t and $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$, we compute the second-order derivatives of $\lambda(p,t)$ w.r.t. t as

$$\left.\frac{\partial^2\lambda(p,t)}{\partial t^2}\right|_{\frac{\partial\lambda(p,t)}{\partial t}=0} = -\frac{\beta_t\lambda(p,t)(\Lambda - \lambda(p,t))(\mu - \lambda(p,t))}{\beta_r\rho t\lambda(p,t)(\Lambda - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) + \omega_\theta\Lambda} < 0.$$

The above implies that, $\forall p$, $\Pi(p,t)$ is a unimodal function of t with a unique optimal solution $t^*(p)$ that can be obtained by solving the following equation for t: $\beta_r\rho(\mu - \lambda(p,t))\exp(-(\mu - \lambda(p,t))t) = \beta_t$. That is, there is a unique solution for the first order condition if $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$.

Next, we find the optimal price for the two scenarios: (1) $\mu < \bar{S}\Lambda + \beta_t/(\beta_r\rho)$ and (2) $\mu \geq \bar{S}\Lambda + \beta_t/(\beta_r\rho)$. For scenario (1), $t^*(p) = \underline{t}$, $\lambda(p, t^*(p)) = \lambda(p, \underline{t})$, $q = 1 - \exp(-(\mu - \lambda(p,\underline{t}))\underline{t})$, and $V = \alpha + \beta_r\rho[1 - \exp(-(\mu - \lambda(p,\underline{t}))\underline{t})] + \beta_r\delta_2 - p - \beta_t\underline{t}$. We use $\underline{t}$ to replace t in $\Pi(p,t) = \lambda(p,t,\phi)(p - b\phi - c)$, and compute the first- and second-order derivatives of $\Pi(p)$ w.r.t. p as

$$\frac{\partial\Pi(p)}{\partial p} = \frac{\partial\lambda(p,\underline{t})}{\partial p}(p - b\phi - c) + \lambda(p,\underline{t}) \text{ and } \frac{\partial^2\Pi(p)}{\partial p^2} = \frac{\partial^2\lambda(p,\underline{t})}{\partial p^2}(p - b\phi - c) + 2\frac{\partial\lambda(p,\underline{t})}{\partial p}.$$

Partially differentiating V once w.r.t. p, we find

$$\begin{aligned}
\frac{\partial V}{\partial p} &= -1 - \underline{t}\beta_r\rho\exp(-(\mu - \lambda(p,\underline{t}))\underline{t})\frac{\partial\lambda(p,\underline{t})}{\partial p}, \\
\frac{\partial^2 V}{\partial p^2} &= -\underline{t}\beta_r\rho\exp(-(\mu - \lambda(p,\underline{t}))\underline{t})\frac{\partial^2\lambda(p,\underline{t})}{\partial p^2} - \underline{t}^2\beta_r\rho\exp(-(\mu - \lambda(p,\underline{t}))\underline{t})\left(\frac{\partial\lambda(p)}{\partial p}\right)^2.
\end{aligned}$$

Similarly, we compute the first- and second-order derivatives of K (replacing t by $\underline{t}$) w.r.t. p as

$$\begin{cases}
\dfrac{\partial K}{\partial p} = & \dfrac{\omega_\theta\Lambda}{\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))}\dfrac{\partial\lambda(p,\underline{t})}{\partial p}, \\
\dfrac{\partial^2 K}{\partial p^2} = & \dfrac{\omega_\theta\Lambda}{\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))}\dfrac{\partial^2\lambda(p,\underline{t})}{\partial p^2} + \omega_\theta\left[-\dfrac{1}{(\lambda(p,\underline{t}))^2} + \dfrac{1}{(\Lambda - \lambda(p,\underline{t}))^2}\right]\left(\dfrac{\partial\lambda(p,\underline{t})}{\partial p}\right)^2.
\end{cases}$$

Solving $\partial V/\partial p = \partial K/\partial p$ gives

$$\frac{\partial \lambda(p,\underline{t})}{\partial p} = -\frac{\lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t}))}{\beta_r \underline{t} \rho \lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t})) \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \Lambda}.$$

We also solve $\partial^2 V/\partial p^2 = \partial^2 K/\partial p^2$ to find

$$\frac{\partial^2 \lambda(p,\underline{t})}{\partial p^2} = \left(\frac{\partial \lambda(p,\underline{t})}{\partial p}\right)^3 \left\{\underline{t}^2 \beta_r \rho \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \left[\frac{1}{(\Lambda - \lambda(p,\underline{t}))^2} - \frac{1}{(\lambda(p,\underline{t}))^2}\right]\right\}.$$

Using $\partial\lambda(p)/\partial p$ and $\partial^2\lambda(p)/\partial p^2$, we can rewrite $\partial\Pi(p)/\partial p$ and $\partial^2\Pi(p)/\partial p^2$ as

$$\frac{\partial \Pi(p)}{\partial p} = -\lambda(p,\underline{t}) \frac{(\Lambda - \lambda(p,\underline{t}))(p - b\phi - c) - \beta_r \underline{t} \rho \lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t})) \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) - \omega_\theta \Lambda}{\beta_r \underline{t} \rho \lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t})) \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \Lambda}.$$

The first order condition (i.e., $\partial\Pi(p)/\partial p = 0$) is $(\Lambda - \lambda(p,\underline{t}))(p - b\phi - c) = \beta_r \underline{t} \rho \lambda(p,\underline{t})(\Lambda - \lambda(p,\underline{t})) \exp(-(\mu - \lambda(p,\underline{t}))\underline{t}) + \omega_\theta \Lambda$ and, at any point where $\partial\Pi(p)/\partial p = 0$, we have that

$$\begin{aligned}
\left.\frac{\partial^2 \Pi(p)}{\partial p^2}\right|_{\frac{\partial \Pi(p)}{\partial p}=0} &= \frac{\partial^2 \lambda(p,\underline{t})}{\partial p^2}(p - b\phi - c) + 2\frac{\partial \lambda(p,\underline{t})}{\partial p} \\
&= \frac{\partial \lambda(p,\underline{t})}{\partial p} \left\{ \frac{(\lambda(p,\underline{t}))^2 \underline{t}^2 \beta_r \rho \exp(-(\mu - \lambda(p,\underline{t}))\underline{t})}{p - c} \right. \\
&\quad \left. + \frac{\omega_\theta (\lambda(p,\underline{t}))^2}{(p - b\phi - c)(\Lambda - \lambda(p,\underline{t}))^2} + \frac{2(p - b\phi - c) - \omega_\theta}{p - b\phi - c} \right\} \\
&< 0.
\end{aligned}$$

Therefore, the optimal price p^* is found as in (5).

Next, we find the optimal price for scenario (ii). As $t^*(p)$ is the unique solution of $\beta_r \rho(\mu - \lambda(p,t)) \exp(-(\mu - \lambda(p,t))t) = \beta_t$, we let $\lambda(p) \equiv \lambda(p, t^*(p))$ and rewrite the delivery reliability as $q = 1 - \exp(-(\mu - \lambda(p))t^*(p)) = 1 - \beta_t/[\beta_r \rho(\mu - \lambda(p))]$. Then, we can rewrite V as $V = \alpha + \beta_r(\rho + \delta_2) - p - \beta_t/[\mu - \lambda(p)] - \beta_t t^*(p)$. Substituting $t^*(p)$ into the firm's profit function gives $\Pi(p) \equiv \Pi(t^*(p), p) = \lambda(p)(p - b\phi - c)$. Partially differentiating V once w.r.t. p, we have

$$\frac{\partial V}{\partial p} = -1 - \frac{\beta_t}{(\mu - \lambda(p))^2} \frac{\partial \lambda(p)}{\partial p} - \beta_t \frac{\partial t^*(p)}{\partial p}.$$

To find $\partial t^*(p)/\partial p$, we take the first-order partial derivative of two sides in (6) w.r.t. p, and solve the resulting equation for $\partial t^*(p)/\partial p$ as

$$\frac{\partial t^*(p)}{\partial p} = \frac{t^*(p)(\mu - \lambda(p)) - 1}{(\mu - \lambda(p))^2} \frac{\partial \lambda(p)}{\partial p}.$$

We then rewrite $\partial V/\partial p$ as

$$\frac{\partial V}{\partial p} = -1 - \frac{\beta_t t^*(p)}{\mu - \lambda(p)} \frac{\partial \lambda(p)}{\partial p},$$

and also compute

$$\frac{\partial^2 V}{\partial p^2} = -\frac{\beta_t t^*(p)}{\mu - \lambda(p)} \frac{\partial^2 \lambda(p)}{\partial p^2} - \frac{2\beta_t t^*(p)(\mu - \lambda(p)) - \beta_t}{(\mu - \lambda(p))^3} \left(\frac{\partial \lambda(p)}{\partial p}\right)^2.$$

Similarly, we compute the first- and second-order derivatives of $K = V_e + \omega_\theta[\ln(\lambda(p,t)) - \ln(\Lambda - \lambda(p,t))]$ w.r.t. p as

$$\begin{cases} \dfrac{\partial K}{\partial p} = \dfrac{\omega_\theta \Lambda}{\lambda(p)(\Lambda - \lambda(p))} \dfrac{\partial \lambda(p)}{\partial p}, \\ \dfrac{\partial^2 K}{\partial p^2} = \dfrac{\omega_\theta \Lambda}{\lambda(p)(\Lambda - \lambda(p))} \dfrac{\partial^2 \lambda(p)}{\partial p^2} + \omega_\theta \left[-\dfrac{1}{(\lambda(p))^2} + \dfrac{1}{(\Lambda - \lambda(p))^2} \right] \left(\dfrac{\partial \lambda(p)}{\partial p} \right)^2. \end{cases}$$

Solving $\partial V/\partial p = \partial K/\partial p$ gives

$$\frac{\partial \lambda(p)}{\partial p} = -\frac{\lambda(p)(\Lambda - \lambda(p))(\mu - \lambda(p))}{\omega_\theta \Lambda(\mu - \lambda(p)) + \beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p))}.$$

We also solve $\partial^2 V/\partial p^2 = \partial^2 K/\partial p^2$ to find

$$\frac{\partial^2 \lambda(p)}{\partial p^2} = \left(\frac{\partial \lambda(p)}{\partial p} \right)^3 \left[\frac{\beta_t}{(\mu - \lambda(p))^2} \left(2t^*(p) - \frac{1}{\mu - \lambda(p)} \right) - \frac{\omega_\theta}{(\lambda(p))^2} + \frac{\omega_\theta}{(\Lambda - \lambda(p))^2} \right].$$

Using $\partial\lambda(p)/\partial p$ and $\partial^2\lambda(p)/\partial p^2$, we can rewrite $\partial\Pi(p)/\partial p$ and $\partial^2\Pi(p)/\partial p^2$ as

$$\frac{\partial \Pi(p)}{\partial p} = \lambda(p) \frac{\beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p)) + \omega_\theta \Lambda(\mu - \lambda(p)) - (\mu - \lambda(p))(\Lambda - \lambda(p))(p - b\phi - c)}{\omega_\theta \Lambda(\mu - \lambda(p)) + \beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p))}.$$

The first order condition (i.e., $\partial\Pi(p)/\partial p = 0$) is $\beta_t t^*(p)\lambda(p)(\Lambda - \lambda(p)) + \omega_\theta\Lambda(\mu - \lambda(p)) = (\mu - \lambda(p))(\Lambda - \lambda(p))(p - b\phi - c)$ and, at any point where $\partial\Pi(p)/\partial p = 0$,

$$\begin{aligned} \left.\frac{\partial^2 \Pi(p)}{\partial p^2}\right|_{\frac{\partial \Pi(p)}{\partial p}=0} &= \frac{\partial^2 \lambda(p)}{\partial p^2}(p - b\phi - c) + 2\frac{\partial \lambda(p)}{\partial p} \\ &= (p - b\phi - c) \left(\frac{\partial \lambda(p)}{\partial p} \right)^3 \left[\frac{\beta_t}{(\mu - \lambda(p))^2} \left(2t^*(p) - \frac{1}{\mu - \lambda(p)} \right) + \frac{\omega_\theta}{(\Lambda - \lambda(p))^2} \right] \\ &\quad + \frac{2(p - b\phi - c) - \omega_\theta}{p - b\phi - c} \frac{\partial \lambda(p)}{\partial p}. \end{aligned}$$

Noting that (7) implies that $\partial\lambda(p)/\partial p < 0$ and that

$$p - b\phi - c = \frac{\beta_t t^*(p)\lambda(p)}{\mu - \lambda(p)} + \frac{\omega_\theta \Lambda}{\Lambda - \lambda(p)} > \frac{\omega_\theta \Lambda}{\Lambda - \lambda(p)} > \omega_\theta,$$

we find that, if $t^*(p) > \mu - \lambda(p)$, then

$$\left.\frac{\partial^2 \Pi(p)}{\partial p^2}\right|_{\frac{\partial \Pi(p)}{\partial p}=0} < 0,$$

which means that $\Pi(p)$ is strictly concave function of p. We thus prove this lemma. ■

Proof of Lemma 2. The first-order derivative of $\lambda^*(\phi) = \lambda(p^*(\phi), t^*(\phi), \phi)$ in (4) w.r.t. ω_θ is

$$\begin{aligned} \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta} &= -\frac{\Lambda \exp[-(V - V_e)/\omega_\theta]}{\{1 + \exp[-(V - V_e)/\omega_\theta]\}^2} \left[-\frac{1}{\omega_\theta} \frac{\partial V}{\partial \omega_\theta} + \frac{V - V_e}{\omega_\theta^2} \right] \\ &= \frac{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))}{\Lambda} \left(\frac{1}{\omega_\theta} \frac{\partial V}{\partial \omega_\theta} - \frac{V - V_e}{\omega_\theta^2} \right). \end{aligned} \tag{19}$$

To find $\partial\lambda^*(\phi)/\partial\omega_\theta$, we differentiate $V = \alpha - p^*(\phi) + \beta_r(\rho q^*(\phi) + \delta_2) - \beta_t t^*(\phi)$ once w.r.t. ω_θ, as

$$\frac{\partial V}{\partial\omega_\theta} = -\frac{\partial p^*(\phi)}{\partial\omega_\theta} + \beta_r\rho\frac{\partial q^*(\phi)}{\partial\omega_\theta} - \beta_t\frac{\partial t^*(\phi)}{\partial\omega_\theta},$$

where $\partial q^*(\phi)/\partial\omega_\theta$ is given as in (8). We differentiate both sides of the equation $\beta_r\rho(\mu-\lambda(p,t))\exp(-(\mu-\lambda(p,t))t) = \beta_t$ in (6) once w.r.t. ω_θ, and obtain

$$\beta_r\rho\exp(-(\mu-\lambda^*(\phi))t^*(\phi))\left[\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}[t^*(\phi)(\mu-\lambda^*(\phi))-1] - (\mu-\lambda^*(\phi))^2\frac{\partial t^*(\phi)}{\partial\omega_\theta}\right] = 0.$$

Because $\beta_r\rho\exp(-(\mu-\lambda^*(\phi))t^*(\phi)) > 0$, we have

$$(\mu-\lambda^*(\phi))^2\frac{\partial t^*(\phi)}{\partial\omega_\theta} = [t^*(\phi)(\mu-\lambda^*(\phi))-1]\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta},$$

or,

$$\frac{\partial t^*(\phi)}{\partial\omega_\theta} = \frac{t^*(\phi)(\mu-\lambda^*(\phi))-1}{(\mu-\lambda^*(\phi))^2}\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta},$$

which implies that $\partial t^*(\phi)/\partial\omega_\theta$ and $\partial\lambda^*(\phi)/\partial\omega_\theta$ have the same sign.

We compute the first-order derivative of $p^*(\phi)$ in (7) w.r.t. ω_θ as

$$\frac{\partial p^*(\phi)}{\partial\omega_\theta} = b\frac{\partial\phi}{\partial\omega_\theta} + \frac{\beta_t\lambda^*(\phi)}{\mu-\lambda^*(\phi)}\frac{\partial t^*(\phi)}{\partial\omega_\theta} + \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{\beta_t t^*(\phi)\mu}{(\mu-\lambda^*(\phi))^2} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right].$$

Together with the definition of ω_θ this yields $\partial\phi/\partial\omega_\theta = -\omega_\theta/[(\beta_r\delta_1)^2\phi]$ and allows us to rewrite $\partial p^*(\phi)/\partial\omega_\theta$ as

$$\frac{\partial p^*(\phi)}{\partial\omega_\theta} = -\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} + \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{\beta_t\lambda^*(\phi)[t^*(\phi)(\mu-\lambda^*(\phi))-1]}{(\mu-\lambda^*(\phi))^3} + \frac{\beta_t t^*(\phi)\mu}{(\mu-\lambda^*(\phi))^2} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right],$$

which indicates that if $\partial\lambda^*(\phi)/\partial\omega_\theta \le 0$, then $\partial p^*(\phi)/\partial\omega_\theta < 0$; but, if $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$, then $\partial p^*(\phi)/\partial\omega_\theta$ may be positive or may be negative.

Using the expressions of $\partial p^*(\phi)/\partial\omega_\theta$, $\partial q^*(\phi)/\partial\omega_\theta$, and $\partial t^*(\phi)/\partial\omega_\theta$, we find

$$\frac{\partial V}{\partial\omega_\theta} = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\frac{2\beta_t t^*(\phi)\mu(\mu-\lambda^*(\phi)) - \beta_t\lambda^*(\phi)}{(\mu-\lambda^*(\phi))^3} + \frac{\omega_\theta\Lambda}{(\Lambda-\lambda^*(\phi))^2}\right]. \tag{20}$$

We rewrite (19) to obtain

$$\frac{\partial V}{\partial\omega_\theta} = \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\frac{\omega_\theta\Lambda}{\lambda^*(\phi)(\Lambda-\lambda^*(\phi))} + \frac{V-V_e}{\omega_\theta}, \tag{21}$$

and using the RHS of the above equation to replace $\partial V/\partial\omega_\theta$ in (20) yields

$$\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}\left[\beta_t\frac{2t^*(\phi)\mu(\mu-\lambda^*(\phi)) - \lambda^*(\phi)}{(\mu-\lambda^*(\phi))^3} + \frac{\omega_\theta\Lambda^2}{\lambda^*(\phi)(\Lambda-\lambda^*(\phi))^2}\right] = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V-V_e}{\omega_\theta}, \tag{22}$$

where $2t^*(\phi)\mu(\mu-\lambda^*(\phi)) - \lambda^*(\phi) > 0$, because $t(\mu-\lambda^*(\phi)) > 1$. It thus follows that the sign of

$\partial\lambda^*(\phi)/\partial\omega_\theta$ depends on the sign of

$$\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V - V_e}{\omega_\theta}.$$

We also differentiate the firm's expected profit $\Pi^*(\phi) = \Pi(p^*(\phi), t^*(\phi), \phi)$ once w.r.t. ω_θ:

$$\begin{aligned}
\frac{\partial\Pi^*(\phi)}{\partial\omega_\theta} &= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}(p^*(\phi) - b\phi - c) + \lambda^*(\phi)\left(\frac{\partial p^*(\phi)}{\partial\omega_\theta} - b\frac{\partial\phi}{\partial\omega_\theta}\right) \\
&= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta}(p^*(\phi) - b\phi - c) + \lambda^*(\phi)\frac{\partial p^*(\phi)}{\partial\omega_\theta} + \lambda^*(\phi)\frac{b\omega_\theta}{\beta_r\phi} \\
&= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta} \times \left\{p^*(\phi) - b\phi - c + \frac{\beta_t(\lambda^*(\phi))^2[t^*(\phi)(\mu - \lambda^*(\phi)) - 1]}{(\mu - \lambda^*(\phi))^3}\right. \\
&\left. + \frac{\beta_t t^*(\phi)\mu\lambda^*(\phi)}{(\mu - \lambda^*(\phi))^2} + \frac{\omega_\theta\Lambda\lambda^*(\phi)}{(\Lambda - \lambda^*(\phi))^2}\right\} \\
&= \frac{\partial\lambda^*(\phi)}{\partial\omega_\theta} \times \left\{\frac{\beta_t(\lambda^*(\phi))^2[t^*(\phi)(\mu - \lambda^*(\phi)) - 1]}{(\mu - \lambda^*(\phi))^3}\right. \\
&\left. + \frac{\beta_t t^*(\phi)\mu\lambda^*(\phi)}{(\mu - \lambda^*(\phi))^2} + \frac{\beta_t t^*(\phi)\lambda^*(\phi)}{\mu - \lambda^*(\phi)} + \frac{\omega_\theta\Lambda^2}{(\Lambda - \lambda^*(\phi))^2}\right\},
\end{aligned}$$

which shows that, as the terms in the curly bracket are all positive, $\partial\Pi^*(\phi)/\partial\omega_\theta$ and $\partial\lambda^*(\phi)/\partial\omega_\theta$ have an identical sign. Because, as shown above, the sign of $\partial\lambda^*(\phi)/\partial\omega_\theta$ is the same as the sign of $b\omega_\theta/[(\beta_r\delta_1)^2\phi] - (V - V_e)/\omega_\theta$, we find that, when the inequality in (9) is satisfied, $\partial\lambda^*(\phi)/\partial\omega_\theta \geq 0$ and thus, $\partial\Pi^*(\phi)/\partial\omega_\theta \geq 0$ and $\partial t^*(\phi)/\partial\omega_\theta \geq 0$. As $\partial\phi/\partial\omega_\theta = -\omega_\theta/[(\beta_r\delta_1)^2\phi] < 0$, we find that $\partial\lambda^*(\phi)/\partial\phi \leq 0$, $\partial\Pi^*(\phi)/\partial\phi \leq 0$, and $\partial t^*(\phi)/\partial\phi \leq 0$, as indicated in the first item of this lemma.

Next, we analyze the impact of ϕ on the expected ratings. Using (2), we find $E(R^*) = \rho q^*(\phi) + E(X) = \rho q^*(\phi) + \delta_1\gamma(1 - \phi) + \delta_2$, and compute

$$\frac{\partial E(R^*)}{\partial\omega_\theta} = -\frac{\beta_t}{\beta_r\rho(\mu - \lambda^*(\phi))^2}\frac{\partial\lambda^*(\phi)}{\partial\omega_\theta} + \frac{\gamma\omega_\theta}{\beta_r^2\delta_1\phi}.$$

Thus, if the inequality in (9) is not satisfied, then $\partial\lambda^*(\phi)/\partial\omega_\theta < 0$ and as $\partial\phi/\partial\omega_\theta < 0$, $\partial\lambda^*(\phi)/\partial\phi = \partial\lambda^*(\phi)/\partial\omega_\theta \times \partial\omega_\theta/\partial\phi > 0$ and $\partial E(R^*)/\partial\phi = \partial E(R^*)/\partial\omega_\theta \times \partial\omega_\theta/\partial\phi < 0$. Otherwise, $\partial\lambda^*(\phi)/\partial\phi < 0$ and $\partial E(R^*)/\partial\phi$ may be non-negative, which depends on the comparison between $-\partial\lambda^*(\phi)/\partial\omega_\theta \times \beta_t/[\rho(\mu - \lambda^*(\phi))^2]$ and $\gamma\omega_\theta/(\beta_r\delta_1\phi)$.

Moreover, according to (20), we find that if $\partial\lambda^*(\phi)/\partial\phi \geq 0$, then we must have $\partial V/\partial\phi < 0$. Then, using (21), we have $V > V_e$. We thus prove this lemma. ■

Proof of Theorem 1. We denote the difference between the LHS and RHS of (9) by

$$h(\phi) \equiv \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V - V_e}{\omega_\theta}.$$

Because $\lim_{\phi\to 0} h(\phi) = +\infty$, we consider the following two cases only:

1. **Case 1**: $\forall\phi \in [0, 1]$, $h(\phi) \geq 0$ or the condition in (9) is always satisfied. In this case, the firm's expected profit is decreasing in ϕ and thus, $\phi^* = 0$.
2. **Case 2**: there exists one point $\phi_0 \in [0, 1)$ such that $h(\phi_0) < 0$ but $h(\phi) \geq 0$ for $\phi \in [0, \phi_0)$. That is, there exists a sufficiently small, positive number ϕ_0 such that $h(\phi) \geq 0$ for $\phi \in [0, \phi_0)$.

As we already have the optimal solution for Case 1, we next conduct our analysis for Case 2.

Note that, for Case 2, $\phi_0 \neq 1$ because, otherwise, if $\phi_0 = 1$, then $h(\phi) \geq 0$, $\forall \phi \in [0,1]$ and thus Case 2 is identical to Case 1. Hence, for Case 2, $h(\phi) < 0$ at point $\phi = \phi_0 \neq 1$. This implies that $V > V_e$, because $b\omega_\theta/[(\beta_r\delta_1)^2\phi] > 0$. We differentiate $h(\phi)$ once w.r.t. ω_θ and find

$$\frac{\partial h(\phi)}{\partial \omega_\theta} = \frac{b\omega_\theta^2}{(\beta_r\delta_1)^4\phi^3} + \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))}, \tag{23}$$

where an expression for $\partial\lambda^*(\phi)/\partial\omega_\theta$ is given as in (19) in the proof of Lemma 2. We consider the following two possibilities.

1. If, at point $\phi = \phi_0$, $\partial\lambda^*(\phi)/\partial\omega_\theta \leq 0$, then $\partial h(\phi)/\partial\omega_\theta > 0$ when $\phi = \phi_0$.
2. If, at point $\phi = \phi_0$, $\partial\lambda^*(\phi)\partial\omega_\theta > 0$, we need to analyze the RHS of (23). We find from equation (22) in the proof of Lemma 2 that

$$\frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\left[\beta_t \frac{2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi)}{(\mu - \lambda^*(\phi))^3} + \frac{\omega_\theta \Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2}\right] = \frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V - V_e}{\omega_\theta},$$

   which can be rewritten as

$$\frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2} = \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\beta_t}{\omega_\theta}\frac{2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi)}{(\mu - \lambda^*(\phi))^3} + \frac{V - V_e}{\omega_\theta^2} > 0,$$

   where the last inequality follows the conditions: (i) $V \geq V_e$, (ii) $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$, and (iii) $2t^*(\phi)\mu(\mu - \lambda^*(\phi)) - \lambda^*(\phi) > 0$ because $t(\mu - \lambda^*(\phi)) > 1$ and $2\mu > \lambda$.

In addition, as $\partial\lambda^*(\phi)/\partial\omega_\theta > 0$ and $\Lambda > \Lambda - \lambda^*(\phi)$, we have

$$\frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))} > \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda^2}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))^2} > 0.$$

Thus, we find from (23) that

$$\frac{\partial h(\phi)}{\partial \omega_\theta} = \frac{b\omega_\theta^2}{(\beta_r\delta_1)^4\phi^3} + \frac{b}{(\beta_r\delta_1)^2\phi} - \frac{\partial \lambda^*(\phi)}{\partial \omega_\theta}\frac{\Lambda}{\lambda^*(\phi)(\Lambda - \lambda^*(\phi))} > 0.$$

The above results indicate that, at point $\phi = \phi_0$, $h(\phi)$ is increasing in ω_θ, or, $h(\phi)$ is decreasing in ϕ because $\partial\phi/\partial\omega_\theta < 0$, i.e.,

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\phi_0} < 0. \tag{24}$$

We claim that $h(\phi) < 0$ for all values of ϕ in $[\phi_0, 1]$. *Suppose* this is not the case. Then, let $\hat{\phi} = \min\{\phi : \phi \in (\phi_0, 1), h(\phi) \geq 0\}$. Then, because of continuity of $h(\phi)$, there must exist a $\tilde{\phi} \in (\phi_0, \hat{\phi})$ such that

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\tilde{\phi}} \geq 0.$$

However, this *cannot* happen because, by definition, $h(\tilde{\phi}) < 0$, from which we can conclude that

$$\left.\frac{\partial h(\phi)}{\partial \phi}\right|_{\phi=\tilde{\phi}} < 0,$$

along similar lines of proof that we derived (24). This proves the claim.

It follows that, if the firm's expected profit $\Pi^*(\phi)$ is increasing in ϕ when $\phi = \phi_0$, then $\Pi^*(\phi)$

is increasing in ϕ for all values of ϕ in $[\phi_0, 1]$. Furthermore, if $\Pi^*(\phi)$ increases with ϕ at point $\phi' \in [0, 1]$, then $\Pi^*(\phi)$ is increasing in any value of $\phi \in (\phi', 1]$. We can thus conclude that it is impossible for $\Pi^*(\phi)$ to be a unimodal function with a single maximum point. That is, $\Pi^*(\phi)$ cannot first monotonically increase until $\phi = \phi' \in [0, 1)$ and then monotonically decrease when $\phi > \phi'$.

As $\lim_{\phi\to 0} h(\phi) > 0$, $\Pi^*(\phi)$ is a monotonically decreasing when ϕ is sufficiently small in the range $(0, 1)$. Hence, $\Pi^*(\phi)$ (for $\phi \in [0, 1]$) has two possible shapes: (1) $\Pi^*(\phi)$ is monotonically decreasing in ϕ, and (2) $\Pi^*(\phi)$ is a reverse unimodal function with a single minimum point that first monotonically decreases until $\phi = \phi' \in [0, 1]$ and then monotonically increases when $\phi > \phi'$. Thus, the optimal decision ϕ^* must be one of end points (0 and 1). Specifically, since $h(\phi)$ is always decreasing in ϕ, we find that, if $h(1) \geq 0$, then $h(\phi)|_{\phi\in[0,1]} \geq 0$ and thus, $\Pi^*(\phi)$ is decreasing in $\phi \in [0, 1]$. That is, if $h(1) \geq 0$, then the firm's optimal decision is $\phi^* = 0$. Let $V_1 \equiv V|_{\phi=1}$. Because

$$h(1) = \lim_{\phi\to 1}\left(\frac{b\omega_\theta}{(\beta_r\delta_1)^2\phi} - \frac{V - V_e}{\omega_\theta}\right) = \frac{b\omega_Y}{(\beta_r\delta_1)^2} - \frac{V_1 - V_e}{\omega_Y},$$

we conclude that if the condition in (12) is satisfied, then $h(1) \geq 0$ and $\phi^* = 0$.

Otherwise, if the condition in (12) does not hold, i.e., $h(1) < 0$, then there exists one point $\phi_0 \in [0, 1]$ such that $h(\phi) \geq 0$ for $\phi \in (0, \phi_0]$ but $h(\phi) < 0$ for $\phi \in (\phi_0, 1]$. For this case, $\Pi^*(\phi)$ decreases with ϕ over the range $(0, \phi_0]$ but $\Pi^*(\phi)$ increases with ϕ over the range $(\phi_0, 1]$. Hence, the optimal decision is $\phi^* = 1$ if $\Pi^*(1) \geq \Pi^*(0)$ and $\phi^* = 0$ otherwise. This proves the theorem. ■

Proof of Corollary 1. We analyze the impact of β_t for $\phi^* = 1$ and $\phi^* = 0$. For the case $\phi^* = 1$, we differentiate the two sides of the expression (6) at $t_1^* \equiv t^*|_{\phi^*=1}$ once w.r.t. β_t, and obtain

$$\frac{\partial t_1^*}{\partial\beta_t} = \frac{\partial\lambda_1^*}{\partial\beta_t}\left(\frac{t_1^*}{\mu - \lambda_1^*} - \frac{1}{(\mu - \lambda_1^*)^2}\right) - \frac{1}{\beta_t(\mu - \lambda_1^*)}, \tag{25}$$

where $\lambda_1^* \equiv \lambda^*|_{\phi^*=1}$. Then, differentiating λ_1^* once w.r.t. β_t yields

$$\frac{\partial\lambda_1^*}{\partial\beta_t} = \frac{\lambda_1^*}{\omega_Y}\frac{\Lambda - \lambda_1^*}{\Lambda}\left[-\frac{\partial p_1^*}{\partial\beta_t} + \beta_r\rho\frac{\partial q_1^*}{\partial\beta_t} - t_1^* - \beta_t\frac{\partial t_1^*}{\partial\beta_t}\right], \tag{26}$$

where $p_1^* \equiv p^*|_{\phi^*=1}$ and $q_1^* \equiv q^*|_{\phi^*=1}$. Taking the first-order derivative of q_1^* w.r.t. β_t gives

$$\frac{\partial q_1^*}{\partial\beta_t} = -(1 - q_1^*)\left[-(\mu - \lambda_1^*)\frac{\partial t_1^*}{\partial\beta_t} + t_1^*\frac{\partial\lambda_1^*}{\partial\beta_t}\right].$$

Using (25), we reduce the above expression of $\partial q_1^*/\partial\beta_t$ to

$$\frac{\partial q_1^*}{\partial\beta_t} = -(1 - q_1^*)\left[\frac{\partial\lambda_1^*}{\partial\beta_t}\frac{1}{\mu - \lambda_1^*} + \frac{1}{\beta_t}\right]. \tag{27}$$

Differentiating p_1^* w.r.t. β_t gives

$$\begin{aligned}\frac{\partial p_1^*}{\partial\beta_t} &= \frac{\beta_t\lambda_1^* t_1^*}{\mu - \lambda_1^*}\left(\frac{1}{\beta_t} + \frac{1}{\mu - \lambda_1^*}\frac{\partial\lambda_1^*}{\partial\beta_t}\right) + \frac{\beta_t\lambda_1^*}{\mu - \lambda_1^*}\frac{\partial t_1^*}{\partial\beta_t} + \frac{\beta_t t_1^*}{\mu - \lambda_1^*}\frac{\partial\lambda_1^*}{\partial\beta_t} + \frac{\Lambda\omega_Y}{(\Lambda - \lambda_1^*)^2}\frac{\partial\lambda_1^*}{\partial\beta_t}\\ &= \frac{\lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\partial t_1^*}{\partial\beta_t}\frac{\beta_t\lambda_1^*}{\mu - \lambda_1^*} + \frac{\partial\lambda_1^*}{\partial\beta_t}\left[\frac{\beta_t t_1^*\mu}{(\mu - \lambda_1^*)^2} + \frac{\Lambda\omega_Y}{(\Lambda - \lambda_1^*)^2}\right].\end{aligned} \tag{28}$$

Using (25), (27), and (28), we rewrite (26) as

$$\begin{aligned}\frac{\partial \lambda_1^*}{\partial \beta_t} &= -\frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\Lambda\omega_Y}\left\{\frac{\partial \lambda_1^*}{\partial \beta_t}\left[\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}+\frac{\beta_r\rho(1-q_1^*)+\beta_t t_1^*}{\mu-\lambda_1^*}\right]\right.\\ &\left.+\frac{\beta_r}{\beta_t}\rho(1-q_1^*)+\left(1+\frac{\partial \lambda_1^*}{\partial \beta_t}\frac{\beta_t}{\mu-\lambda_1^*}\right)\left(1+\frac{\lambda_1^*}{\mu-\lambda_1^*}\right)\left(t_1^*-\frac{1}{\mu-\lambda_1^*}\right)\right\}. \qquad (29)\end{aligned}$$

If $\partial\lambda_1^*/\partial\beta_t \geq 0$, then the RHS of (29) is negative because $1 > q_1^*$ and $t_1^* \geq 1/(\mu-\lambda_1^*)$. As a result, the equation in (29) does not hold. That is, $\partial\lambda_1^*/\partial\beta_t \geq 0$ is *impossible*. It thus follows that $\partial\lambda_1^*/\partial\beta_t < 0$. According to (25), we also have $\partial t_1^*/\partial\beta_t < 0$. However, we find from (27) that $\partial q_1^*/\partial\beta_t \geq 0$ if

$$\frac{\partial \lambda_1^*}{\partial \beta_t}\frac{1}{\mu-\lambda_1^*}+\frac{1}{\beta_t}\leq 0, \text{ or, } -\frac{\partial \lambda_1^*/\lambda_1^*}{\partial \beta_t/\beta_t}\geq\frac{\mu-\lambda_1^*}{\lambda_1^*},$$

and $\partial q_1^*/\partial\beta_t < 0$ otherwise. Moreover, $\partial p_1^*/\partial\beta_t \geq 0$ if

$$\frac{\lambda_1^* t_1^*}{\mu-\lambda_1^*}\geq -\frac{\partial t_1^*}{\partial \beta_t}\frac{\beta_t\lambda_1^*}{\mu-\lambda_1^*}-\frac{\partial \lambda_1^*}{\partial \beta_t}\left[\frac{\beta_t t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right].$$

and $\partial p_1^*/\partial\beta_t < 0$ otherwise.

We take the first-order derivative of $\Pi_1^* \equiv \Pi^*|_{\phi^*=1}$ w.r.t. β_t and find

$$\frac{\partial \Pi_1^*}{\partial \beta_t}=\frac{(\lambda_1^*)^2 t_1^*}{\mu-\lambda_1^*}+\frac{\partial \lambda_1^*}{\partial \beta_t}\left[p_1^*-b-c+\frac{\beta_t\lambda_1^* t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\lambda_1^*\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]+\frac{\partial t_1^*}{\partial \beta_t}\frac{\beta_t(\lambda_1^*)^2}{\mu-\lambda_1^*}.$$

Hence, $\partial\Pi_1^*/\partial\beta_t \geq 0$ if

$$\frac{(\lambda_1^*)^2 t_1^*}{\mu-\lambda_1^*}\geq -\frac{\partial \lambda_1^*}{\partial \beta_t}\left[p_1^*-b-c+\frac{\beta_t\lambda_1^* t_1^*\mu}{(\mu-\lambda_1^*)^2}+\frac{\Lambda\lambda_1^*\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]-\frac{\partial t_1^*}{\partial \beta_t}\frac{\beta_t(\lambda_1^*)^2}{\mu-\lambda_1^*},$$

and $\partial\Pi_1^*/\partial\beta_t < 0$ otherwise. To derive meaning implications from the above results, we simplify them to find that $\partial p_1^*/\partial\beta_t < 0$ and $\partial\Pi_1^*/\partial\beta_t < 0$ if

$$\frac{\lambda_1^* t_1^*}{\mu-\lambda_1^*}\leq -\frac{\partial t_1^*}{\partial \beta_t}\frac{\beta_t\lambda_1^*}{\mu-\lambda_1^*}, \text{ or, } -\frac{\partial t_1^*/t_1^*}{\partial \beta_t/\beta_t}\geq 1.$$

Following a similar process, we obtain the impacts of β_t for $\phi^* = 0$. The proof is complete. ■

Proof of Corollary 2. For $\phi^* = 1$, we differentiate both sides of expression (6) w.r.t. β_r and obtain

$$\frac{\partial t_1^*}{\partial \beta_r}(\mu-\lambda_1^*)=\frac{1}{\beta_r}+\frac{\partial \lambda_1^*}{\partial \beta_r}\left(t_1^*-\frac{1}{\mu-\lambda_1^*}\right). \qquad (30)$$

We then compute the first-order derivative of λ_1^* w.r.t. β_r and find

$$\frac{\partial \lambda_1^*}{\partial \beta_r}=\frac{\lambda_1^*}{\omega_Y}\frac{\Lambda-\lambda_1^*}{\Lambda}\left(-\frac{\partial p_1^*}{\partial \beta_r}+\rho q_1^*+\delta_2+\beta_r\rho\frac{\partial q_1^*}{\partial \beta_r}-\beta_t\frac{\partial t_1^*}{\partial \beta_r}\right). \qquad (31)$$

Taking the first-order derivative of q_1^* in (1) w.r.t. β_r and using (6) and (30) to simplify it yields

$$\frac{\partial q_1^*}{\partial \beta_r}=\frac{\beta_t}{\beta_r\rho}\left(\frac{\partial t_1^*}{\partial \beta_r}-\frac{t_1^*}{\mu-\lambda_1^*}\frac{\partial \lambda_1^*}{\partial \beta_r}\right)=\frac{\beta_t}{\beta_r\rho(\mu-\lambda_1^*)}\left(\frac{1}{\beta_r}-\frac{\partial \lambda_1^*}{\partial \beta_r}\frac{1}{\mu-\lambda_1^*}\right),$$

which is non-negative when

$$\frac{1}{\beta_r} - \frac{\partial \lambda_1^*}{\partial \beta_r}\frac{1}{\mu-\lambda_1^*} \geq 0, \text{ or, } \frac{\partial \lambda_1^*/\lambda_1^*}{\partial \beta_r/\beta_r} \leq \frac{\mu-\lambda_1^*}{\lambda_1^*}.$$

Also, we differentiate p_1^* once w.r.t. β_r and obtain

$$\frac{\partial p_1^*}{\partial \beta_r} = \frac{\beta_t \lambda_1^*}{\beta_r(\mu-\lambda_1^*)^2} + \frac{\partial \lambda_1^*}{\partial \beta_r}\left[\frac{\beta_t\lambda_1^* t_1^*}{\mu-\lambda_1^*} + \frac{2\beta_t\lambda_1^* t_1^*}{(\mu-\lambda_1^*)^2} - \frac{\beta_t\lambda_1^*}{(\mu-\lambda_1^*)^3} + \frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right].$$

We then use the above results to rewrite $\partial\lambda_1^*/\partial\beta_r$ in (31) as

$$\begin{aligned}\frac{\partial \lambda_1^*}{\partial \beta_r} \quad = \quad & \frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}\left\{\rho+\delta_2-\frac{\beta_t\mu}{\beta_r(\mu-\lambda_1^*)^2} - \frac{\partial \lambda_1^*}{\partial \beta_r}\left[\frac{\beta_t\lambda_1^* t_1^*}{\mu-\lambda_1^*} + \frac{\beta_t t_1^*}{\mu-\lambda_1^*}\right.\right. \\ & \left.\left. + \frac{2\beta_t\lambda_1^* t_1^*}{(\mu-\lambda_1^*)^2} - \frac{\beta_t\lambda_1^*}{(\mu-\lambda_1^*)^3} + \frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right]\right\}. \end{aligned} \tag{32}$$

If $\rho+\delta_2 \leq \beta_t\mu/[\beta_r(\mu-\lambda_1^*)^2]$, or,

$$q_1^* \leq \frac{\beta_t\lambda_1^* - \beta_r\delta_2(\mu-\lambda_1^*)^2}{\beta_t\mu - \beta_r\delta_2(\mu-\lambda_1^*)^2},$$

then $\partial\lambda_1^*/\partial\beta_r \leq 0$. Otherwise, $\partial\lambda_1^*/\partial\beta_r$ may be positive. In particular, when the expected delivery time $1/(\mu-\lambda^*)$ is negligibly short, i.e., $1/(\mu-\lambda^*) \to 0$, in (32) we can remove all terms involving $1/(\mu-\lambda^*)$. As a result, (32) can be reduced to

$$\frac{\partial \lambda_1^*}{\partial \beta_r} = \frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}\left[\rho+\delta_2 - \frac{\partial \lambda_1^*}{\partial \beta_r}\frac{\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right] = \frac{\lambda_1^*(\Lambda-\lambda_1^*)}{\omega_Y\Lambda}(\rho+\delta_2) - \frac{\partial \lambda_1^*}{\partial \beta_r}\frac{\lambda_1^*}{\Lambda-\lambda_1^*},$$

which can be rewritten as

$$\frac{\partial \lambda_1^*}{\partial \beta_r} = \frac{\lambda_1^*(\Lambda-\lambda_1^*)^2}{\omega_Y\Lambda^2}(\rho+\delta_2) > 0.$$

We take the first-order derivative of Π_1^* w.r.t. β_r and find

$$\frac{\partial \Pi_1^*}{\partial \beta_r} = \frac{\beta_t(\lambda_1^*)^2}{\beta_r(\mu-\lambda_1^*)^2} + \frac{\partial \lambda_1^*}{\partial \beta_r}\left[p_1^* - b - c + \frac{\beta_t(\lambda_1^*)^2 t_1^*}{\mu-\lambda_1^*} + \frac{2\beta_t(\lambda_1^*)^2 t_1^*}{(\mu-\lambda_1^*)^2} - \frac{\beta_t(\lambda_1^*)^2}{(\mu-\lambda_1^*)^3} + \frac{\lambda_1^*\Lambda\omega_Y}{(\Lambda-\lambda_1^*)^2}\right],$$

which means $\partial\Pi_1^*/\partial\beta_r \geq 0$ if $\partial\lambda_1^*/\partial\beta_r \geq 0$.

The analyses for $\phi^*=0$ are similar to those for $\phi^*=1$. We differentiate $\lambda_0^* \equiv \lambda^*|_{\phi^*=0}$ once w.r.t. β_r and get

$$\begin{aligned}\frac{\partial \lambda_0^*}{\partial \beta_r} \quad = \quad & \frac{\lambda_0^*(\Lambda-\lambda_0^*)}{\Lambda\sqrt{(\beta_r\delta_1)^2+\omega_Y^2}}\left\{\rho+\delta_2-\frac{\beta_t\mu}{\beta_r(\mu-\lambda_0^*)^2} - \frac{\partial \lambda_0^*}{\partial \beta_r}\left[\frac{\beta_t\lambda_0^* t_0^*}{\mu-\lambda_0^*} + \frac{\beta_t t_0^*}{\mu-\lambda_0^*} + \frac{2\beta_t\lambda_0^* t_0^*}{(\mu-\lambda_0^*)^2}\right.\right. \\ & \left.\left. - \frac{\beta_t\lambda_0^*}{(\mu-\lambda_0^*)^3} + \frac{\Lambda\sqrt{(\beta_r\delta_1)^2+\omega_Y^2}}{(\Lambda-\lambda_0^*)^2} + \frac{\beta_r\delta_1(V_0-V_e)}{(\beta_r\delta_1)^2+\omega_Y^2}\right]\right\} - \frac{\lambda_0^*\beta_r\delta_1}{(\beta_r\delta_1)^2+\omega_Y^2}, \end{aligned}$$

where $t_0^* \equiv t^*|_{\phi^*=0}$. Taking the first-order derivative of $p_0^* \equiv p^*|_{\phi^*=0}$ w.r.t. β_r gives

$$\frac{\partial p_0^*}{\partial \beta_r} = \frac{\beta_t \lambda_0^*}{\mu - \lambda_0^*}\frac{\partial t_0^*}{\partial \beta_r} + \frac{\partial \lambda_0^*}{\partial \beta_r}\left[\frac{\beta_t t_0^*}{\mu - \lambda_0^*} + \frac{\beta_t \lambda_0^* t_0^*}{(\mu - \lambda_0^*)^2} + \frac{\Lambda\sqrt{(\beta_r \delta_1)^2 + \omega_Y^2}}{(\Lambda - \lambda_0^*)^2}\right] + \frac{\Lambda \beta_r \delta_1}{(\Lambda - \lambda_0^*)\sqrt{(\beta_r \delta_1)^2 + \omega_Y^2}}.$$

Other derivatives are similar to those for $\phi^* = 1$. Using the arguments for $\phi^* = 1$, we produce the results for $\phi^* = 0$ as shown in this corollary. ■

Proof of Corollary 3. For $\phi^* = 1$, we differentiate the two sides of the expression for t_1^* once w.r.t. μ, and obtain

$$\frac{\partial t_1^*}{\partial \mu}(\mu - \lambda_1^*)^2 = [1 - t_1^*(\mu - \lambda_1^*)]\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

Then, differentiating λ_1^* once w.r.t. μ yields

$$\frac{\partial \lambda_1^*}{\partial \mu} = \frac{\lambda_1^*}{\omega_Y}\frac{\Lambda - \lambda_1^*}{\Lambda}\left(-\frac{\partial p_1^*}{\partial \mu} + \beta_r \rho \frac{\partial q_1^*}{\partial \mu} - \beta_t \frac{\partial t_1^*}{\partial \mu}\right), \tag{33}$$

Taking the first-order derivative of q_1^* w.r.t. μ gives

$$\frac{\partial q_1^*}{\partial \mu} = \frac{\beta_t}{\beta_r \rho}\left[\frac{\partial t_1^*}{\partial \mu} + \frac{t_1^*}{\mu - \lambda_1^*}\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right)\right] = \frac{\beta_t}{\beta_r \rho (\mu - \lambda_1^*)^2}\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

Differentiating p_1^* w.r.t. μ gives

$$\frac{\partial p_1^*}{\partial \mu} = \frac{\beta_t t_1^*}{\mu - \lambda_1^*}\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(\frac{1}{\mu - \lambda_1^*} - 2t_1^*\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right) + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \mu}.$$

We then rewrite $\partial \lambda_1^*/\partial \mu$ in (33) as

$$\frac{\partial \lambda_1^*}{\partial \mu} = \frac{\lambda_1^*(\Lambda - \lambda_1^*)}{\omega_Y \Lambda} - \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left[\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right) + \frac{\beta_t t_1^*}{\mu - \lambda_1^*}\left(1 - 2\frac{\partial \lambda_1^*}{\partial \mu}\right)\right],$$

where the RHS is positive if the LHS (i.e., $\partial \lambda_1^*/\partial \mu \leq 0$). That is, $\partial \lambda_1^*/\partial \mu > 0$. Moreover, if $\partial \lambda_1^*/\partial \mu \geq 1$, the RHS of the above equation is negative, which breaks the equation up. Therefore, we have $0 < \partial \lambda_1^*/\partial \mu < 1$. As a result, $\partial t_1^*/\partial \mu < 0$ and $\partial q_1^*/\partial \mu > 0$.

We take the first-order derivative of Π_1^* w.r.t. μ and find

$$\frac{\partial \Pi_1^*}{\partial \mu} = \left[p_1^* - b - c + \frac{\beta_t \lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \lambda_1^* \omega_Y}{(\Lambda - \lambda_1^*)^2}\right]\frac{\partial \lambda_1^*}{\partial \mu} + \frac{\beta_t (\lambda_1^*)^2}{(\mu - \lambda_1^*)^2}\left(\frac{1}{\mu - \lambda_1^*} - 2t_1^*\right)\left(1 - \frac{\partial \lambda_1^*}{\partial \mu}\right).$$

We can conclude that for $\phi^* = 1$, if the expected delivery time is sufficiently short, then both $\partial p_1^*/\partial \mu$ and $\partial \Pi_1^*/\partial \mu$ are positive.

The analysis for $\phi^* = 0$ is analogous to that for $\phi^* = 1$. The corollary is thus proved. ■

Proof of Corollary 4. For $\phi^* = 1$, we differentiate the two sides of the expression for t_1^* once w.r.t. ω_Y, and have

$$\frac{\partial t_1^*}{\partial \omega_Y}(\mu - \lambda_1^*)^2 = [t_1^*(\mu - \lambda_1^*) - 1]\frac{\partial \lambda_1^*}{\partial \omega_Y}.$$

Then, differentiating λ_1^* once w.r.t. ω_Y yields

$$\frac{\partial \lambda_1^*}{\partial \omega_Y} = \frac{\lambda_1^*}{\omega_Y}\frac{(\Lambda - \lambda_1^*)}{\Lambda}\left(-\frac{\partial p_1^*}{\partial \omega_Y} + \beta_r \rho \frac{\partial q_1^*}{\partial \omega_Y} - \beta_t \frac{\partial t_1^*}{\partial \omega_Y} + \frac{V_1 - V_e}{\omega_Y}\right). \tag{34}$$

Taking the first-order derivative of q_1^* w.r.t. ω_Y gives

$$\frac{\partial q_1^*}{\partial \omega_Y} = -\frac{\beta_t}{\beta_r \rho(\mu - \lambda_1^*)}\left[-(\mu - \lambda_1^*)\frac{\partial t_1^*}{\partial \omega_Y} + \frac{\partial \lambda_1^*}{\partial \omega_Y} t_1^*\right] = -\frac{\beta_t}{\beta_r \rho(\mu - \lambda_1^*)^2}\frac{\partial \lambda_1^*}{\partial \omega_Y}.$$

Differentiating p_1^* w.r.t. ω_Y gives

$$\frac{\partial p_1^*}{\partial \omega_Y} = \frac{\Lambda}{\Lambda - \lambda_1^*} + \frac{\partial \lambda_1^*}{\partial \omega_Y}\left[\frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{\beta_t t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\right].$$

Using the above to simplify $\partial \lambda_1^*/\partial \omega_Y$ gives

$$\begin{aligned}\frac{\partial \lambda_1^*}{\partial \omega_Y} &= \frac{\lambda_1^*}{\omega_Y}\frac{(\Lambda - \lambda_1^*)}{\Lambda}\left\{\beta_r \delta_2 + \frac{V_1 - V_e}{\omega_Y} - \frac{\Lambda}{\Lambda - \lambda_1^*}\right.\\ &\left. -\frac{\partial \lambda_1^*}{\partial \omega_Y}\left[\frac{\beta_t \lambda_1^*}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{2\beta_t t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \omega_Y}{(\Lambda - \lambda_1^*)^2}\right]\right\}.\end{aligned} \tag{35}$$

We can then use the expression of λ_1^* to find that

$$\frac{\Lambda}{\Lambda - \lambda_1^*} = \frac{\Lambda}{\lambda_1^*}\exp\left(\frac{V_1 - V_e}{\omega_Y}\right) > \exp\left(\frac{V_1 - V_e}{\omega_Y}\right),$$

which implies

$$\frac{V_1 - V_e}{\omega_Y} > \frac{\Lambda}{\Lambda - \lambda_1^*}.$$

Therefore, $\partial \lambda_1^*/\partial \omega_Y \geq 0$, because the equation in (35) does not hold if $\partial \lambda_1^*/\partial \omega_Y < 0$. Hence, $\partial t_1^*/\partial \omega_Y \geq 0$, $\partial p_1^*/\partial \omega_Y \geq 0$, and $\partial q_1^*/\partial \omega_Y \leq 0$. We also differentiate Π_1^* once w.r.t. ω_Y and have

$$\frac{\partial \Pi_1^*}{\partial \omega_Y} = \frac{\Lambda \lambda_1^*}{\Lambda - \lambda_1^*} + \frac{\partial \lambda_1^*}{\partial \omega_Y}\left[p_1^* - b - c + \frac{\beta_t (\lambda_1^*)^2}{(\mu - \lambda_1^*)^2}\left(2t_1^* - \frac{1}{\mu - \lambda_1^*}\right) + \frac{\beta_t \lambda_1^* t_1^*}{\mu - \lambda_1^*} + \frac{\Lambda \lambda_1^* \omega_Y}{(\Lambda - \lambda_1^*)^2}\right],$$

which is non-negative because $\partial \lambda_1^*/\partial \omega_Y \geq 0$.

The results for $\phi^* = 0$ are the same as those for $\phi^* = 1$, because $\partial\sqrt{(\beta_r \delta_1)^2 + \omega_Y^2}/\partial \omega_Y = \omega_Y/\sqrt{(\beta_r \delta_1)^2 + \omega_Y^2} > 0$. This corollary is thus proved. ■

Appendix E Additional Numerical Results

We extend our numerical analyses of the *baseline* scenario to consider a broader set of model parameters and to test the robustness of the results to the assumption that $Z \sim C(\phi)$ (which implies that $X \sim C(\phi)$ because $\delta_1 = 1$ and $\delta_2 = 0$). We generate data to characterize the relationship between ϕ and Π^* for different values of the parameters b, c, d, α, ρ, V_e, and the parameters already considered in the main text (β_r, β_t, μ, and ω_Y). This time, we evaluate the parameters at the values $\{0, 5, 9\}$ because of computational singularities with the value 10 when considering alternative distributions.

The results for $Z \sim C(\phi)$ appear in Figure A and replicate those presented in the main text for parameters d, β_r, β_t, μ, and ω_Y. The numerical results for the additional parameters indicate that profits are higher when costs (b and c) are lower, when the perceived quality of the firm's offering (α) is high, when the perceived value of balking (V_e) is low, and when the effect of delivery reliability on delivery trust (ρ) is strong. The intuition behind the results for b, c, V_e, and α is self-evident. The intuition behind the result for ρ is similar to the intuition behind the result for β_r because both parameters determine the weight of delivery reliability information in the utility function in similar ways. In addition, we note that variation in the values of β_r and ρ results in the largest variation in profits among all parameters. This suggests that β_r and ρ are the most important determinants of profitability. This result is confirmed by an elasticity analyses (available upon request).

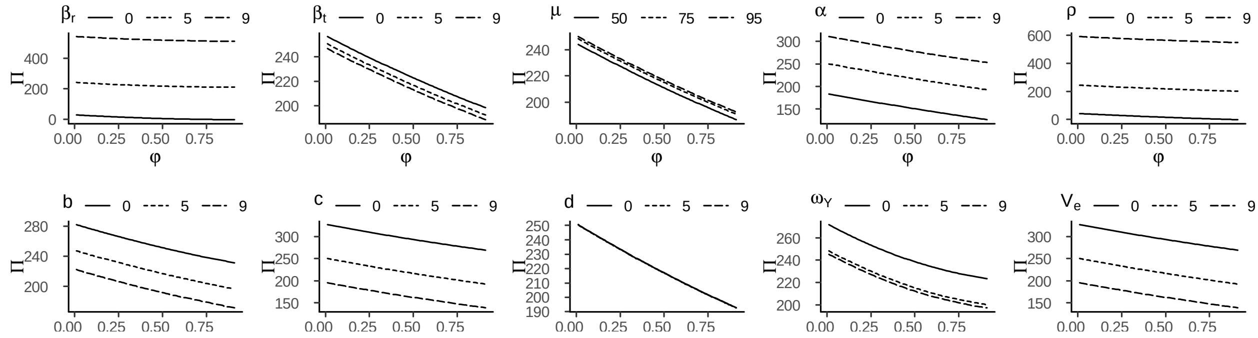

Figure A: Sensitivity analyses under the assumption that $Z \sim C(\phi)$.

The $C(\phi)$ distribution allows us to obtain closed form solutions and analytical results but it is of interest to determine whether results hold for other popular distributions. Because closed-form solutions do not exist for distributions other than the $C(\phi)$ distribution, we perform these analyses numerically. We consider the cases in which $Z \sim$ *Gumbel*, $Z \sim \log normal$, and $Z \sim Normal$ because these distributions are widely used in individual-level models of demand. We use these alternative distributions to replicate the sensitivity analyses so as to draw easily interpretable results while considering a wide range of parameter values. To ensure comparability to the base results obtained with the $C(\phi)$ distribution, we restrict the mean and variance of these alternative distributions to be equal to those of the $C(\phi)$ distribution for each value of ϕ considered.

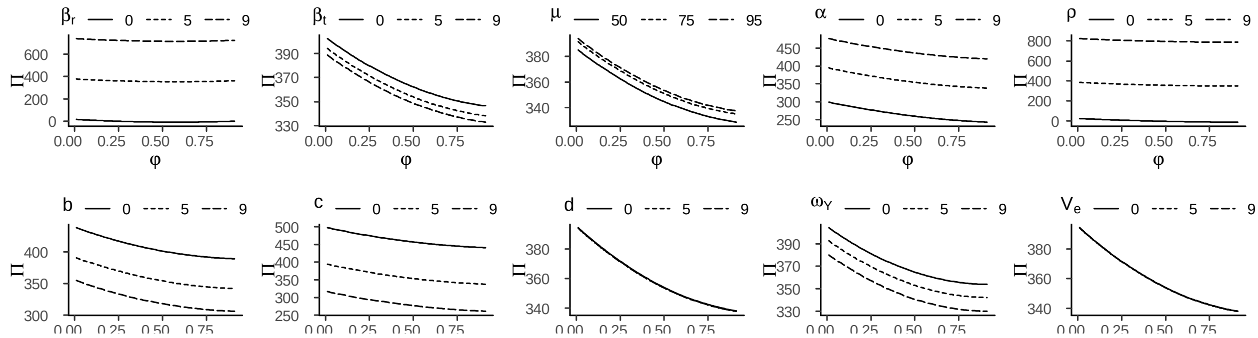

Figure B: Sensitivity analyses under the assumption that $Z \sim$ *Gumbel*.

Results for the Gumbel distribution appear in Figure B and for the lognormal and normal distributions in Figures C and D, respectively. All three figures indicate that the shapes of the profit curves are very similar across distributions. Most importantly, the main result that only $\phi = 0$ or $\phi = 1$ can be optimal holds regardless of the distributional assumption.

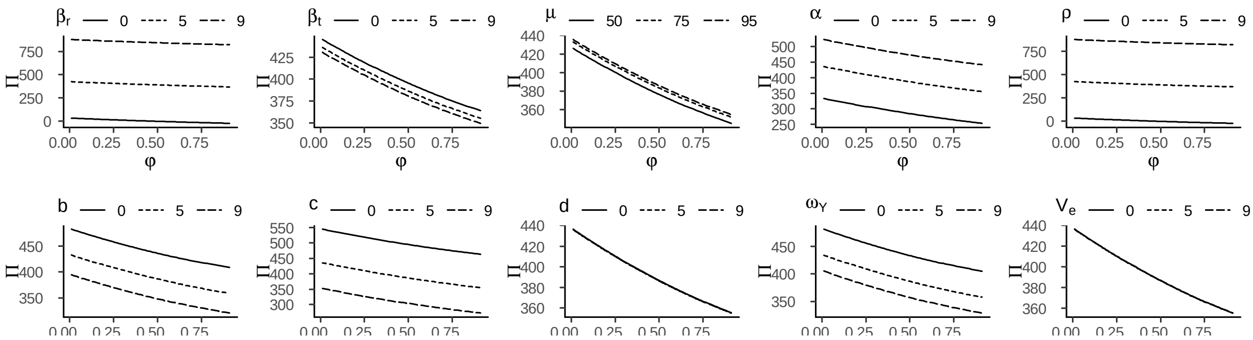

Figure C: Sensitivity analyses under the assumption that $Z \sim \log normal$.

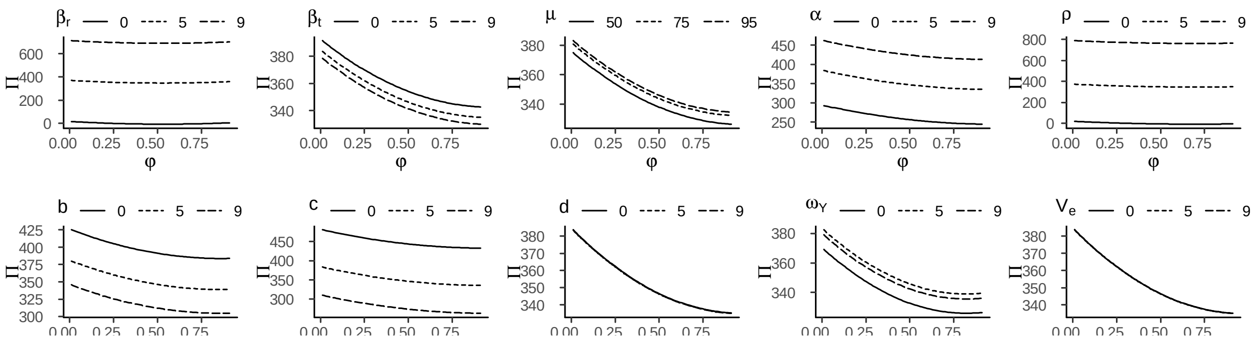

Figure D: Sensitivity analyses under the assumption that $Z \sim Normal$.