Optimal Retail Pricing, Interest Rate, and Interest Allocation Ratio Decisions in an Online Platform-Assisted Financing System[a, b]

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Abstract

We examine an online financing system involving a platform, a bank, and a retailer, in which the bank makes the optimal interest rate decision before the bank-platform negotiation for the interest allocation ratio (scenario 1) or after the interest allocation ratio negotiation (scenario 2). We find that the retailer's sales and profits in scenario 1 are higher than those in scenario 2. Moreover, if the referral fee rate increases, the system-wide profit and the system efficiency for scenario 1 increase but those for scenario 2 decrease.

Keywords: Platform financing; Nash bargaining; sequential-move game.

1 Introduction

The growth of e-commerce has induced an increasing number of small and medium-sized enterprises (SMEs) to use online platforms for their operational activities (Wang et al. (Wang, Fan and Yin 2019) and Liu et al. (Liu, Shang, Wu and Chen 2020)). Recent years have witnessed that the COVID-19 pandemic severely impacts SMEs, leaving them with depleted cash reserves. Accessing credit has become significantly challenging due to a high demand in the credit extension sector. Those SMEs may suffer from a shortage of working capital, as it is usually difficult for them to obtain a loan from traditional banks due to their lack of preferred collateral, credit information, and profitability instability (Tang et al. (Tang, Yang and Wu 2018)). Gong et al. (Gong, Liu, Liu and Ren 2020), Zhen et al. (Zhen, Shi, Li and Zhang 2020), and Rath et al. (Rath, Basu, Mandal and Paul 2021) have exposed that online platforms can fully grasp the information about their retail members (e.g., consumer reviews, transaction history, and purchase records). This enables the platforms to provide historical data to banks for their loan assessments and also assists the SMEs with a shortage of working capital in meeting the bank's loan requirements (Tuna and Zhu 2018). Hence, as Yan et al. (Yan, Liu, Xu and He 2020) and Yan et al. (Yan, Zhang, Xu and Gao 2021) discussed, the collaboration between a platform and a financing service provider is helpful to online loan issues.

To exemplify the bank-platform cooperation for the loans to retailers, we consider `DHgate.com` which is the most extensive online B2B platform in China and has been cooperating with some banks

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such as the China Construction Bank and the China Merchants Bank. Any qualified retailer who often conducts transactions at `DHgate.com` can apply for a loan. `DHgate.com` converts the platform data into a credit line approved by the bank. The bank then uses the retailer's transaction data and credit history in the platform as the basis for credit, completes independent approval, and grants loans. Qualified retailers can obtain loans from banks with no collateral or deposit. During the cooperation, the platform only provides a preliminary review of the retailer's information, and the bank conducts a final loan assessment with the retailer's information. This practice reveals that the platform's sharing of the retailer's information with the bank is the *prerequisite* for the bank-platform cooperation. That is, in this paper, the platform assumes the role of information revealing because, otherwise, the bank cannot assess the retailer's qualification and thus cannot cooperate with the platform for online loans.

Under the influence of COVID-19, many American banks and high technology firms have jumped into the ring to increase lending availability. Amazon has recently collaborated with Lendistry, a lending platform specializing in small business loans, to provide "platform assisted financing" services by sharing the SME transaction data with Lendistry when SMEs apply for the loan on Amazon's platform (Leffert 2022). Amazon has also partnered with Marcus Goldman Sachs–a leading financial services provider–for providing SMEs the loans totaling up to $1 million (Son 2020). The partnership between Amazon and Goldman presents numerous advantages for both parties. Offering financing options to SMEs enhances the appeal of Amazon's marketplace, particularly during a time when the online retail market is emerging rapidly. As for Goldman Sachs, it has made significant progress in lending services within a short period (Pymnts 2020). Meanwhile, Goldman also takes the plunge into cooperation with Apple to launch lending services to the App Store's users (Goldman 2019).

In this paper, we consider a three-firm decision-making problem involving a bank (B), an e-commerce platform (E), and a capital-constrained retailer (R). The bank extends loan services to the retailer who caters to price-sensitive consumers in the market. The retailer pays a loan interest to the bank which, in turn, shares a portion of the interest with the platform based on an interest allocation ratio, similar to the practice that Amazon secures a percentage of Marcus Goldman Sachs's interest income (Shevlin 2020). The interest allocation ratio is important to achieving stability of the collaboration between the bank and the platform. It is interesting to explore the optimal interest allocation ratio which should reflect the bargaining power between the platform and the bank. Nonetheless, very few researchers have considered the bargaining issues in the platform-assisted financing field. For example, Ma et al. (Ma, Dai and Li 2022) assumed that the platform solely determines an allocation ratio to divide the loan amount (rather than the loan interest) between the platform and the bank. In contrast, we follow the common industry practices (by, e.g., Amazon) to compute the allocation of the loan interest. We also analyze the impact of decision sequences on optimal decisions and profits.

Moreover, the retailer does not have any other sales channel but only sells its products on the platform. As the platform aims to help the retailer obtain the loan from the bank, the platform should allow the retailer to expose products to all potential consumers on the platform. Thus, in this paper, we do not consider the platform's role in user traffic (i.e., allowing the retailer to sell to a larger market) and instead use a logit function to characterize the aggregate demand from all potential consumers on the platform.

The rest of this paper is organized as follows. In Section 2 we discuss decision sequences and profit functions. In Section 3, we analyze two three-stage scenarios to find optimal solutions. In Section 4, we compare our analytical results for the two scenarios to find managerial implications. We relegate our proofs to online Appendix A.

2 Decision Sequences and Profit Functions

We investigate a three-firm platform-assisted financing system, in which a retailer purchases a product at unit acquisition cost c and sells it to consumers at a retail price p on an online platform. The p-dependent market demand for the product, denoted by $D(p)$, is specified in a logit function form as $D(p) = \exp(\lambda)/\left[1 + \exp(\lambda)\right]$, where $\lambda \equiv \alpha - \beta p$. Here, $\alpha > 0$ denotes the total potential demand and $\beta > 0$ represents the marginal impact of the retail price. The logit demand function has been widely used in publications relevant to online platforms; see, for example, Bapna et al. (Bapna, Goes, Wei and Zhang 2011) and Sato (Sato 2021).

Therefore, the retailer's total cost is $cD(p)$ and the retailer's profit (resulting from sales) is $(p - c)D(p)$. As an SME with a lack of working capital, the retailer secures a loan in the amount of $cD(p)$ from a bank on the platform. For the loan, the retailer pays to the bank an interest amounting to $icD(p)$ where i represents the interest rate charged by the bank. The platform usually receives a portion of the bank's interest income as its commission fee. For instance, Amazon secures a percentage of Marcus Goldman Sachs's interest income (Shevlin 2020). We use $\delta \in [0, 1)$ to denote the rate of the platform's commission fee from the bank on the interest amount. The rate δ is also known as the interest allocation ratio. As the retailer's loan interest is $icD(p)$, the platform and the bank obtain $\delta icD(p)$ and $ic(1 - \delta)D(p)$, respectively.

The above indicates the importance of the interest rate and interest allocation ratio decisions. Accordingly, the bank and the platform determine interest rate i and interest allocation ratio δ, respectively. As in practice, the bank can independently decide on the interest rate whereas the bank and the platform usually bargain over the interest allocation ratio. Since the bank engages in the two decision-making issues, the bank can choose the sequence of these two decisions; that is, the bank can make a decision on whether to (1) first determine the interest rate and then negotiate the allocation ratio with the platform or to (2) first bargain the allocation ratio with the platform and then make its interest rate decision. Next, we specify scenarios 1 and 2 in which the retailer, as an SME, typically assumes the role of "follower," thus setting a retail price after the bank's and the platform's decisions.

1. In scenario 1, there are three stages. In the first stage, the bank determines the interest rate and then announces it to the platform. In the second stage, the bank and the platform play a

two-person cooperative game to bargain over the interest allocation ratio. In the third stage, the retailer decides on a retail price.

2. In scenario 2, there are also three stages. In the first stage, the bank bargains with the platform over the interest allocation plan. Then, the bank makes an interest rate decision in the second stage. The retailer determines the retail price in the third stage.

If the bank chooses scenario 1, then it acts as the "leader" in the game and thus could find an interest rate to influence the allocation ratio negotiation with an aim to benefit the bank. However, prior to bargaining over the allocation ratio with the bank, the platform can observe the interest rate. The platform can accordingly play a certain negotiation strategy to secure an allocation that may not be desirable by the bank. As the bank has announced the interest rate, it cannot change its interest rate in accordance with the platform's negotiation strategy. Scenario 2 allows the bank to first obtain the negotiated result for the allocation ratio and then utilize this certain information to make the optimal interest rate decision. However, in this scenario, the bank does not hold the "leader" position to impact the interest allocation ratio negotiation. As a result, the platform may play a negotiation strategy to obtain a higher allocation than in scenario 1, which would lead the bank to determine the optimal interest rate that may make the bank achieve a lower profit than in scenario 1.

For both scenarios, in addition to loan interest $icD(p)$ paid to the bank, for each product sold on the platform, the retailer needs to pay a percentage of its revenue to the platform. Such payment is known as a referral fee, for which the percentage, denoted by $g \in [0, 1)$, is called the referral fee rate. Therefore, the retailer pays a referral fee $gpD(p)$ to the platform. In reality, the fee rate at the official website of major e-commerce platforms such as `Amazon.com` (Amazon 2023) is usually constant for each category of products sold by all retailers on different online platforms. This fee rate is announced in advance, and retailers who intend to enter an online platform must comply with this fee rate. This implies that referral fee rate g could be reasonably viewed as a "standard" in the eBusiness industry. For example, referring to the charging standards displayed on the official website of several major e-commerce platforms in the US market, their average fee rate is around 15%. We accordingly consider g as an exogenous variable, similar to Gong et al. (Gong et al. 2020), Zennyo (Zennyo 2020), Yi et al. (Yi, Wang and Chen 2021), and Ma et al. (Ma et al. 2022).

We can thus compute the retailer's, the bank's, and the platform's expected (net) profits as

$$\pi_R(p, i) \equiv \frac{\exp(\lambda)\,(p - c - ic - gp)}{1 + \exp(\lambda)};\ \pi_B(p, i, \delta) \equiv \frac{\exp(\lambda)\,[ic(1 - \delta)]}{1 + \exp(\lambda)};\ \pi_E(p, i, \delta) \equiv \frac{\exp(\lambda)\,(gp + \delta ic)}{1 + \exp(\lambda)}.$$

Since the interest allocation ratio δ is negotiated by the bank and the platform, we use the generalized Nash bargaining scheme (GNBS) (Nash 1950) to characterize the negotiated result. We denote the platform's and the bank's relative bargaining powers by $a \in (0, 1)$ and $1 - a$, respectively. For this negotiation, the disagreement points for the platform and the bank are zero, because neither the bank nor the platform can gain any profit from the online financing system if the bank does not provide its financing service in the platform. Thus, to find the negotiated result for scenario $j = 1, 2$, we need to solve the following problem: $\max_{\delta_j} \Lambda \equiv (\pi_E^j)^a(\pi_B^j)^{1-a}$, subject to $\pi_E^j \geq 0$ and $\pi_B^j \geq 0$, where δ_j is the negotiated interest allocation ratio for scenario j, π_E^j and π_B^j are the platform's and the bank's profits for scenario j, respectively.

3 Analysis of the Online Financing System

Prior to analyzing two (decentralized) scenarios, we investigate the centralized case to find the globally optimal decisions for the three firms, which are later compared with our decentralized results. In the centralized case, the system-wide profit is $\Pi(p) \equiv \pi_R(p, i) + \pi_B(p, i, \delta) + \pi_E(p, i, \delta)$.

Proposition 1 *For the centralized case, the globally-optimal retail price is*

$$p_C^* = \frac{\exp(\lambda_C) + \beta c + 1}{\beta},$$

where λ_C can be uniquely obtained by solving the following equation for λ: $\lambda + \exp(\lambda) = \alpha - \beta c - 1$. We then compute the maximum system-wide profit as $\Pi_C^* \equiv \Pi(p_C^*) = \exp(\lambda_C)/\beta$.

Next, we analyze two three-stage scenarios for the online financing system, starting with scenario 1.

Proposition 2 *Under scenario 1 (i.e., when the bank's interest rate decision takes place before the bank-platform interest allocation negotiation), we can find the unique interest rate and retail price in Stackelberg equilibrium by solving the following equations:*

$$i_1^* \equiv \frac{(1-g)^2 [1 + \exp(\lambda_1)]^2}{\beta c} - g \text{ and } p_1^* \equiv \frac{(1-g)\exp(2\lambda_1) + (3-2g)\exp(\lambda_1) + \beta c + 2 - g}{\beta},$$

where λ_1 can be uniquely obtained by solving the following equation for λ:

$$\lambda = \alpha - (1-g)\exp(2\lambda) - (3-2g)\exp(\lambda) - \beta c - 2 + g.$$

Then, we can uniquely find the negotiated interest allocation ratio as

$$\delta_1^* = \frac{(g-1)(a-g)\exp(2\lambda_1) + \left[-2g^2 + g(a+3) - 2a\right]\exp(\lambda_1) + \beta c g - g^2 - a + 2g}{\beta c g - (1-g)^2 [1 + \exp(\lambda_1)]^2},$$

and calculate the resulting profits for the three firms as

$$\begin{cases} \pi_R^{1*} \equiv \pi_R^1(p_1^*, i_1^*) = \dfrac{(1-g)\exp(\lambda_1)}{\beta}, \\ \pi_B^{1*} \equiv \pi_B^1(p_1^*, i_1^*, \delta_1^*) = \dfrac{(1-a)\exp(\lambda_1)[1 + (1-g)\exp(\lambda_1)]}{\beta}, \\ \pi_E^{1*} \equiv \pi_E^1(p_1^*, i_1^*, \delta_1^*) = \dfrac{a\exp(\lambda_1)[1 + (1-g)\exp(\lambda_1)]}{\beta}, \end{cases}$$

For scenario 1, the system-wide profit is

$$\Pi_D^{1*} \equiv \pi_R^{1*} + \pi_B^{1*} + \pi_E^{1*} = \frac{\exp(\lambda_1)\left[(1-g)\exp(\lambda_1) + 2 - g\right]}{\beta}.$$

We then perform game analysis for scenario 2.

Proposition 3 *Under scenario 2 (i.e., when the bank makes its interest rate decision after the bank-platform interest allocation negotiation), we can uniquely obtain the retail price and interest rate in Stackelberg equilibrium by solving the following equations:*

$$i_2^* \equiv \frac{(1-g)\left[1+\exp(\lambda_2)\right]^2}{\beta c} \text{ and } p_2^* \equiv \frac{(1-g)\left[\exp(2\lambda_2) + 3\exp(\lambda_2) + 2\right] + \beta c}{\beta(1-g)},$$

where λ_2 can be uniquely obtained by solving the following equation for λ:

$$\lambda = \alpha - \exp(2\lambda) - 3\exp(\lambda) - \beta c/(1-g) - 2.$$

Then, we can find the unique negotiated interest allocation ratio as

$$\delta_2^* = \frac{(1-g)(a-g)\exp(2\lambda_2) + (1-g)\left[(a-3)g + 2a\right]\exp(\lambda_2) + g^2(2-a) + g[(a-1)\beta c - 2] + a}{(1-g)^2\left[1+\exp(\lambda_2)\right]^2}.$$

We also compute the resulting profits as

$$\begin{cases} \pi_R^{2*} \equiv \pi_R^2(p_2^*, i_2^*) = \dfrac{(1-g)\exp(\lambda_2)}{\beta}, \\ \pi_B^{2*} \equiv \pi_B^2(p_2^*, i_2^*, \delta_2^*) = \dfrac{\exp(\lambda_2)(1-a)\left[(1-g)\exp(2\lambda_2) - (g^2+g-2)\exp(\lambda_2) + \beta cg + 1 - g^2\right]}{\beta(1-g)\left[1+\exp(\lambda_2)\right]}, \\ \pi_E^{2*} \equiv \pi_E^2(p_2^*, i_2^*, \delta_2^*) = \dfrac{a\exp(\lambda_2)\left[(1-g)\exp(2\lambda_2) - (g^2+g-2)\exp(\lambda_2) + \beta cg + 1 - g^2\right]}{\beta(1-g)\left[1+\exp(\lambda_2)\right]}, \end{cases}$$

The system-wide profit for scenario 2 is

$$\Pi_D^{2*} = \frac{\exp(\lambda_2)}{1+\exp(\lambda_2)}\left[\frac{\exp(2\lambda_2) + 3\exp(\lambda_2) + 2}{\beta} + \frac{cg}{1-g}\right].$$

Propositions 2 and 3 expose that the decision-making sequence significantly affects all firms' optimal decisions and profits.

4 Comparative Analyses

We perform comparison analyses to derive managerial implications regarding which scenario is more favorable to the bank and the platform. This finding is important to understanding whether the two firms have an incentive to engage in scenario 1 or 2.

Proposition 4 *For the online system, we find the following results.*

1. $\lambda_1 > \lambda_2$ and thus, the demand under scenario 1 is always greater than scenario 2.

2. $p_1^* < p_2^*$ but i_1^* (δ_1^*) may be greater or may be smaller than i_2^* (δ_2^*).

3. $\pi_R^{1*} > \pi_R^{2*}$ but π_B^{1*} (π_E^{1*}) may be greater or may be smaller than π_B^{2*} (π_E^{2*}).

Proposition 4 certainly indicates that the retailer charges a lower price ($p_1^* < p_2^*$), achieves higher sales ($\lambda_1 > \lambda_2$), and obtains a greater profit ($\pi_R^{1*} > \pi_R^{2*}$) under scenario 1 than under scenario 2. These definite results can be justified as follows. Under scenario 1, the bank determines its interest rate prior to negotiating the interest allocation ratio with the platform. That is, the ratio decision is the two firms' "best response" to the bank's interest rate decision. As our decision-making problem is one with complete information, the bank has the knowledge of the best-response ratio decision in terms of its interest rate. This helps the bank find the optimal interest rate decision i_1^* that maximizes its profit.

However, under scenario 2, the bank's interest rate decision occurs after the ratio negotiation. Therefore, we can regard the interest rate decision as the bank's best response to the bank-platform ratio negotiation that depends on the two firms' profits as well as their relative bargaining powers. As the ratio negotiation is made by the two firms rather than the bank itself, the ratio decision may not be most desirable by the bank. Hence, for scenario 2, the bank's optimal interest rate i_2^* (as the best response to the ratio decision) may not be most beneficial to the bank, which means that i_2^* may differ from i_1^*, as shown in item 2 of Proposition 4, and the bank may have an incentive to change its interest rate decision under scenario 2 if i_2^* is different from i_1^*. This implies that the interest rate decision for scenario 2 (i.e., i_2^*) could be "rather uncertain" compared to the interest rate decision for scenario 1 (i.e., i_1^*). As the interest rate influences the retailer's profit, the retailer can charge a lower price to attract consumers under scenario 1 but cannot ensure the effectiveness of the lower-price strategy under scenario 2, i.e., $p_1^* < p_2^*$. As consumers are highly sensitive to the retail price, a lower price can increase the demand, i.e., $\lambda_1 > \lambda_2$, which brings about a higher profit to the retailer under scenario 1, i.e., $\pi_R^{1*} > \pi_R^{2*}$, as shown in Proposition 4.

Our findings in Proposition 4 expose that scenario 1 is more conducive to attracting customers because it can reduce uncertainty and allow for a lower-price strategy that can increase the market demand and lead to a higher profit for the retailer. In addition, Proposition 4 shows that the bank's "first-mover" and "second-mover" strategies can generate different outcomes for the platform and the bank.

Next, we compare the system-wide profit in each decentralized scenario (i.e., scenario 1 or 2) with that in the centralized case to assess the efficiency of each decentralized system, which is measured as the ratio of the decentralized system-wide profit to the centralized system-wide profit. That is, for scenario $i = 1, 2$, the efficiency of the corresponding decentralized system is computed as

$$\tau_i = \frac{\Pi_D^{i*}}{\Pi_C^*} = \begin{cases} \dfrac{\exp(\lambda_1)\left[(1-g)\exp(\lambda_1) + 2 - g\right]}{\exp(\lambda_C)}, & \text{if } i = 1; \\ \dfrac{\exp(\lambda_2)\left[\exp(2\lambda_2) + 3\exp(\lambda_2) + 2 + (\beta c g)/(1-g)\right]}{\exp(\lambda_C)\left[1 + \exp(\lambda_2)\right]}, & \text{if } i = 2, \end{cases}$$

where Π_D^{i*} is the system-wide profit in terms of the optimal decisions for scenario i, and Π_C^* represents the system-wide profit in terms of the optimal retail price for the centralized case. As the system-wide profit in any decentralized scenario must be smaller than the centralized system-wide profit, i.e., $\Pi_C^* > \max\{\Pi_D^{1*}, \Pi_D^{2*}\}$, we have $\tau_i \leq 1$, for $i = 1, 2$.

Proposition 5 *Both the system-wide profit in terms of the optimal decisions for scenario 1 (i.e., Π_D^{1*}) and the system efficiency for scenario 1 (i.e., τ_1) increase with referral fee rate g but those for scenario 2 (i.e., Π_D^{2*} and τ_2) decrease with g.*

Proposition 5 implies that, as the impact of referral fee rate g on the system efficiency differs under two scenarios, we cannot determine under which scenario the system efficiency is greater, i.e., the efficiency may be greater or may be smaller under scenario 1 than under scenario 2. In fact, under scenario 1, a proper increase in the value of g benefits the system efficiency, because, although a higher value of g makes the retailer absorb a greater referral fee, it raises the platform's gain and thus may induce the platform to accept a lower interest allocation ratio in the negotiation with the bank. Then, the bank may respond by charging the retailer a smaller interest rate. This could motivate the retailer to acquire more products for sales on the platform and therefore, the system efficiency improves, which means that a higher referral rate makes the decentralized system behave closer to the centralized system. Similarly, using the above argument, we can find the impact of a decrease in g on the system efficiency under scenario 2 as shown in Proposition 5, i.e., a lower referral rate makes the decentralized system closer to the centralized system. We conclude that referral rate g plays a role in the platform-bank interest allocation ratio negotiation, thereby influencing the system performance.

We obtain the insights provided by Proposition 5 in the platform-assisted financing setting. In practice, although the platform-assisted financing system is common, there exist other loan settings which include the "platform credit financing" system and the "bank credit financing" system. In future, it would be interesting to explore our game problem in different loan settings. Moreover, two or more platforms may compete with their financing strategies, and two or more retailers compete for consumers with their pricing strategies on a single platform that provides financing services. As such competitive cases are common in reality, we would investigate our game problem in a competing setting.

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Online Supplements

"**Optimal Retail Pricing, Interest Rate, and Interest Allocation Ratio Decisions in an Online Platform-Assisted Financing System**"

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Appendix A Proofs

Proof of Proposition 1. For the centralized system, the system-wide profit is

$$\Pi(p) \equiv \pi_R(p,i) + \pi_B(p,i,\delta) + \pi_E(p,i,\delta) = \exp(\lambda_C)(p-c)/\left[1+\exp(\lambda_C)\right],$$

where $\lambda_C = \alpha - \beta p$. The first- and second-order derivatives of $\Pi(p)$ w.r.t. p are

$$\begin{aligned}
\frac{\partial \Pi(p)}{\partial p} &= \frac{\exp(2\lambda_C) - [\beta(p-c)-1]\exp(\lambda_C)}{[1+\exp(\lambda_C)]^2}, \\
\frac{\partial^2 \Pi(p)}{\partial p^2} &= -\frac{\beta \exp(\lambda_C)}{[1+\exp(\lambda_C)]^2} < 0.
\end{aligned}$$

Hence, we can solve the first order condition $\partial \Pi(p)/\partial p = 0$ to uniquely obtain the globally-optimal retail price as given in this proposition, and then compute the maximum system-wide profit as in this proposition. ■

Proof of Proposition 2. We compute the first- and second-order derivatives of $\pi_R(p,i)$ w.r.t. p as

$$\begin{aligned}
\frac{\partial \pi_R(p,i)}{\partial p} &= \frac{(1-g)\left\{\exp\left[2(\alpha-\beta p)\right] + \exp(\alpha-\beta p)\right\} + \gamma\exp(\alpha-\beta p)}{[1+\exp(\alpha-\beta p)]^2}, \\
\frac{\partial^2 \pi_R(p,i)}{\partial p^2} &= \frac{\beta\left\{\gamma\left\{\exp\left[2(\alpha-\beta p)\right] - \exp(\alpha-\beta p)\right\} - 2(1-g)\left\{\exp\left[2(\alpha-\beta p)\right] + \exp(\alpha-\beta p)\right\}\right\}}{[1+\exp(\alpha-\beta p)]^3},
\end{aligned}$$

where $\gamma \equiv \beta\left[c(1+i) + p(g-1)\right]$. Since $\lambda_1 = \alpha - \beta p_1$, the retailer's optimal (best response) retail price $p^*(i)$ can be obtained by solving the following equation for p_1:

$$p = \frac{1+\exp(\lambda_1)}{\beta} + \frac{c(1+i)}{1-g}, \tag{1}$$

where λ_1 satisfies the equation:$\lambda_1 = \alpha - \exp(\lambda_1) - 1 - \beta c(1+i)/(1-g)$.

Using (1) and the equation for λ_1, we compute $\gamma = -(1-g)\left[1+\exp(\lambda_1)\right] < 0$ and have

$$\frac{\partial \lambda_1}{\partial i} = -\exp(\lambda_1)\frac{\partial \lambda_1}{\partial i} - \frac{\beta c}{1-g} = \frac{\beta c}{\gamma} < 0 \text{ and } \frac{\partial^2 \lambda_1}{\partial i^2} = -\frac{\beta^2 c^2 \exp(\lambda_1)}{\gamma^2\left[1+\exp(\lambda_1)\right]} < 0.$$

Substituting $p^*(i)$ in (1) into $\partial^2\pi_R(p,i)/\partial p^2$ gives

$$\frac{\partial^2\pi_R(p,i)}{\partial p^2} = -\frac{\beta\left(1-g\right)\exp(\lambda_1)\left\{1+2\exp(\lambda_1)+\exp\left\{2\left[\lambda_1+\beta c\left(1+i\right)/\left(1-g\right)\right]\right\}\right\}}{\left[1+\exp(\lambda_1)\right]^3} < 0.$$

Thus, $\pi_R(p,i)$ is unimodal in p.

For scenario 1, substituting $p^*(i)$ in (1) into $\pi_B(p,i,\delta)$ and $\pi_E(p,i,\delta)$, we have

$$\begin{aligned} \pi_B\left(i,\delta\right) &= \frac{ic(1-\delta)\exp(\lambda_1)}{1+\exp(\lambda_1)}, \\ \pi_E\left(i,\delta\right) &= \frac{\left\{g\left(g-1\right)\exp(\lambda_1)+g^2+g\left\{\beta c\left[i\left(\delta-1\right)-1\right]-1\right\}-\beta ci\delta\right\}\exp(\lambda_1)}{\beta\gamma}. \end{aligned} \tag{2}$$

To obtain the negotiation solution for interest allocation ratio δ, we solve the maximum problem $\max_\delta \wedge\left(i,\delta\right) = (\pi_{_E}\left(i,\delta\right))^a(\pi_{_B}\left(i,\delta\right))^{1-a}$. Given the interest rate, the first- and second-order derivatives of $\wedge\left(i,\delta\right)$ w.r.t. δ are

$$\frac{\partial\wedge\left(i,\delta\right)}{\partial\delta} = \left[\pi_{_B}\left(i,\delta\right)\right]^{1-a}\left[\pi_{_E}\left(i,\delta\right)\right]^a\left\{\frac{a-1}{1-\delta}+\frac{a\beta ci\left(g-1\right)}{g\left(g-1\right)\exp(\lambda_1)+g^2+g\left\{\beta c\left[i\left(\delta-1\right)-1\right]-1\right\}-\beta ci\delta}\right\},$$

and

$$\begin{aligned} \frac{\partial^2\wedge\left(i,\delta\right)}{\partial\delta^2} &= -a\left[\pi_{_B}\left(i,\delta\right)\right]^{-a}\left\{\pi_{_E}\left(i,\delta\right)\right\}^a ic(a-1)\exp(\lambda_1)\left[(\beta c+1)g+\beta ci-g^2\right] \\ &\quad\times\left\{\frac{g^2(1-g)^2\exp(2\lambda_1)+\left[2g(1-g)\exp(\lambda_1)-g^2+(\beta c+1)g+\beta ci\right]}{\left[1+\exp(\lambda_1)\right]\left(\delta-1\right)\left\{(g^2-g)\exp(\lambda_1)+g^2+(-1+\beta c(\delta i-i-1))g-ci\delta\beta\right\}^2}\right\}. \end{aligned}$$

Solving $\partial\wedge\left(i,\delta\right)/\partial\delta=0$ for δ yields

$$\delta^*(i) = \frac{g\left(g-1\right)\left(a-1\right)\exp(\lambda_1)+\left(a-1\right)g^2+g\left[-a\left(1+\beta c\right)+1+\beta c\left(1+i\right)\right]-a\beta ci}{\beta ci\left(g-1\right)}.$$

At the point satisfying $\partial\wedge\left(i,\delta\right)/\partial\delta=0$, we find that

$$\frac{\partial^2\wedge\left(i,\delta\right)}{\partial\delta^2} = -\left\{\frac{\left(a-1\right)\theta\exp(\lambda_1)}{\beta\gamma}\right\}^{-a}\left\{-\frac{a\theta\exp(\lambda_1)}{\beta\gamma}\right\}^a\left\{\frac{\beta c^2i^2\left(1-g\right)\exp(\lambda_1)}{a\theta\left[1+\exp(\lambda_1)\right]}\right\},$$

where $\theta = g\left(1-g\right)\exp(\lambda_1)-g^2+g\left(\beta c+1\right)+\beta ci$ and $\gamma = -\left(1-g\right)\left[1+\exp(\lambda_1)\right]$. Since $\theta>0$ and $\gamma<0$, we have $\partial^2\wedge\left(i,\delta\right)/\partial\delta^2<0$. That is, $\wedge\left(i,\delta\right)$ is unimodal in δ.

Substituting $\delta^*(i)$ into $\pi_B\left(i,\delta\right)$ yields

$$\pi_B\left(i\right) = \frac{\theta\left(1-a\right)\exp(\lambda_1)}{\beta\left(1-g\right)\left[1+\exp(\lambda_1)\right]}.$$

We differentiate $\pi_B(i)$ once and twice w.r.t. i, and have

$$\begin{aligned}\frac{\partial \pi_B(i)}{\partial i} &= -\frac{\exp(\lambda_1)\,(1-a)\,\beta c\,[\theta/\gamma + (1-g)\exp(\lambda_1)+1]}{\beta\gamma\,[1+\exp(\lambda_1)]},\\ \frac{\partial^2 \pi_B(i)}{\partial i^2} &= -\frac{\beta c^2\,(1-a)\exp(\lambda_1)\,[2\,(1-g)\exp(2\lambda_1)+(3-2g)\exp(\lambda_1)+1]}{\gamma^2\,[1+\exp(\lambda_1)]^2}.\end{aligned}$$

Solving $\partial\pi_B(i)/\partial i = 0$, we obtain

$$i_1^* = \frac{(1-g)^2\,[1+\exp(\lambda_1)]^2}{\beta c} - g.$$

Because $2\,(1-g)\exp(2\lambda_1)+(3-2g)\exp(\lambda_1)+1>0$, we have $\partial^2\pi_B(i)/\partial i^2<0$, which means that $\pi_B(i)$ is unimodal in i.

Using i_1^* to replace i in $p_1^*(i)$ and $\delta^*(i)$ gives the retail price and interest allocation ratio in Stackelberg equilibrium as in this proposition. In addition, we can compute the three firms' profits (i.e., π_R^{1*}, π_B^{1*}, and π_E^{1*}) and the system-wide profit $\Pi_D^{1^*} = \pi_R^{1*}+\pi_B^{1*}+\pi_E^{1*}$ as shown in this proposition. ∎

Proof of Proposition 3. For scenario 2, $p^*(i)$ is given as in (1), the bank's best-response function $\pi_B(i,\delta)$ given as in (2). Using $p^*(i)$ we have

$$\lambda_2 = \alpha - \beta p_2 = \alpha - \exp(\lambda_2) - 1 - \frac{\beta c\,(1+i)}{1-g}.$$

Taking the first- and second-order derivatives of λ_2 w.r.t. i at both sides of the above equation, we have

$$\frac{\partial\lambda_2}{\partial i} = -\frac{\beta c}{(1-g)\,[1+\exp(\lambda_2)]} < 0 \text{ and } \frac{\partial^2\lambda_2}{\partial i^2} = -\frac{\beta^2 c^2\exp(\lambda_2)}{\gamma^2\,[1+\exp(\lambda_2)]} < 0.$$

We differentiate $\pi_B(i,\delta)$ once and twice w.r.t. i as

$$\begin{aligned}\frac{\partial \pi_B(i,\delta)}{\partial i} &= \frac{c(1-\delta)\exp(\lambda_2)\,[1+\exp(\lambda_2)]\,[i\,(\partial\lambda_2/\partial i)+1] - ic\,(\partial\lambda_2/\partial i)\,[\exp(\lambda_2)]^2\,(1-\delta)}{[1+\exp(\lambda_2)]^2},\\ \frac{\partial^2 \pi_B(i,\delta)}{\partial i^2} &= -\frac{\exp(\lambda_2)c(\delta-1)}{[1+\exp(\lambda_2)]^3}\{-\exp(2\lambda_2)\,(\partial\lambda_2/\partial i) + i[(\partial^2\lambda_2/\partial i^2) - (\partial\lambda_2/\partial i)^2]\exp(\lambda_2)\\ &\quad + (\partial\lambda_2/\partial i)^2\,i + (\partial^2\lambda_2/\partial i^2)\,i + (\partial\lambda_2/\partial i)\}.\end{aligned}$$

Solving $\partial\pi_B(i,\delta)/\partial i = 0$, we obtain

$$i_2^* = \frac{(1-g)\,[1+\exp(\lambda_2)]^2}{\beta c}.$$

Since $i_2^* > 0$, $(\partial\lambda_2/\partial i) < 0$. At the point i_2^*,

$$\frac{\partial^2 \pi_B(i,\delta)}{\partial i^2} = -\frac{c\,(1-\delta)\left(\partial^2\lambda_2/\partial i^2\right)\exp(\lambda_2)}{(\partial\lambda_2/\partial i)\,[1+\exp(\lambda_2)]} < 0,$$

which implies that $\pi_B(i,\delta)$ is unimodal in i.

Using i_2^* to replace i in $p_2^*(i)$, we have the retail price in Stackelberg equilibrium p_2^* as in this proposition. Using i_2^* to replace i in $\pi_B(i,\delta)$ gives

$$\pi_B(\delta) = \frac{c\,(1-\delta)\,(1-g)\exp(\lambda_2)\,[1+\exp(\lambda_2)]}{\beta};$$

and substituting i_2^* and p_2^* into $\pi_E\,(p,i,\delta)$ yields

$$\begin{aligned}\pi_E\,(\delta) \;&=\; \frac{\exp(\lambda_2)}{\beta(1-g)\,[1+\exp(\lambda_2)]}\{(g-1)[(\delta-1)g-\delta]\exp(2\lambda_2)\\ &\quad +2(g-1)\,[(\delta-3/2)g-\delta]\exp(\lambda_2)+(\delta-2)g^2+(\beta c-2\delta+2)g+\delta\}.\end{aligned}$$

To find the bargaining solution δ, we should solve the maximum problem $\max_\delta \wedge(\delta) = (\pi_{_E}\,(\delta))^a(\pi_{_B}\,(\delta))^{1-a}$. The first- and second-order derivatives of $\wedge(\delta)$ w.r.t. δ are

$$\begin{aligned}\frac{\partial\wedge(\delta)}{\partial\delta} \;&=\; -\frac{(g-1)\,[\pi_B(\delta)]^{-a}\,[\pi_E\,(\delta)]^a\exp(2\lambda_2)}{\pi_E\,(\delta)\,\beta^2(1-g)}\{-(g-1)\,[(\delta-1)g-\delta]\exp(2\lambda_2)\\ &\quad -(g-1)\,[(a+2\delta-3)g+2a-2\delta]\exp(\lambda_2)+(-a-\delta+2)g^2\\ &\quad +(a\beta c-\beta c+2\delta-2)g+a-\delta\},\end{aligned}$$

and

$$\begin{aligned}\frac{\partial^2\wedge(\delta)}{\partial\delta^2} \;&=\; -\frac{a\,(a-1)\exp(3\lambda_2)\,[\pi_B(\delta)]^{-a}\,[\pi_E\,(\delta)]^a}{\beta^3\,(1-\delta)\,(1-g)\,[\pi_E\,(\delta)]^2\,[1+\exp(\lambda_2)]}\\ &\quad \times\{-2(g-1)\left[\delta cg-\left(g^3\right)/2-\left(5g^2\right)/2+3\right]\exp(2\lambda_2)+2(g^3-3g+2)\exp(3\lambda_2)\\ &\quad +(1-g)^2\exp(4\lambda_2)+(\beta cg-g^2+1)\left[2(-g^2-g+2)\exp(\lambda_2)+\beta cg-g^2+1\right]\}.\end{aligned}$$

Solving $\partial\wedge(\delta)/\partial\delta = 0$ for δ yields

$$\delta_2^* = \frac{(1-g)(a-g)\exp(2\Omega_2)+(1-g)\,[(a-3)g+2a]\exp(\lambda_2)+(2-a)g^2+(a\beta c-\beta c-2)g+a}{(1-g)^2\,[1+\exp(\lambda_2)]^2}.$$

As the point δ_2^*, we find that

$$\frac{\partial^2\wedge(\delta)}{\partial\delta^2} = -\frac{c^2\exp(\lambda_2)\beta\,[1+\exp(\lambda_2)]\,(1-g)\,[\pi_B(\delta)]^{-a}\,[\pi_E\,(\delta)]^a}{a\,(\partial\lambda_2/\partial i)\,\{(g(1-g)\,(\partial\lambda_2/\partial i)-\beta c)\exp(\lambda_2)+g(\beta c+1-g)\,(\partial\lambda_2/\partial i)-\beta c\}},$$

where $\partial\lambda_2/\partial i = -\beta c/[(1-g)(1+\exp(\lambda_2))] < 0$. Since $[g(1-g)(\partial\lambda_2/\partial i)-\beta c]\exp(\lambda_2)+g(\beta c+1-g)(\partial\lambda_2/\partial i)-\beta c < 0$, $\partial^2\wedge(\delta)/\partial\delta^2 < 0$ and thus, $\wedge(\delta)$ is unimodal in δ.

Using the Stackelberg equilibrium, we can compute the three firms' and the system-wide profits as shown in this proposition. ■

Proof of Proposition 4. Using Propositions 2 and 3, we compute

$$\pi_R^{1*} - \pi_R^{2*} = \frac{(1-g)\left[\exp(\lambda_1) - \exp(\lambda_2)\right]}{\beta}.$$

Moreover, we calculate $\lambda_1 - \lambda_2 = \hat{\lambda} \equiv -(1-g)\exp(2\lambda_1) - (3-2g)\exp(\lambda_1) + \exp(2\lambda_2) + 3\exp(\lambda_2) + \beta c/(1-g) - \beta c + g$, which is decreasing in λ_1. Replacing λ_1 with λ_2 in $\hat{\lambda}$, $\hat{\lambda}$ is reduced to $\eta \equiv g\left[1+\exp(\lambda_2)\right]^2 + (\beta cg)/(1-g) > 0$. As $\exp(\lambda_i) > 1$ (for $i = 1, 2$) and $\eta > 0$, we find that $\lambda_1 > \lambda_2$ and $\pi_R^{1*} > \pi_R^{2*}$. In addition, since $\lambda_1 > \lambda_2$, we have $\lambda_1 - \lambda_2 < \eta$ because $\hat{\lambda}$ decreases with λ_1. Because $D(p) = \exp(\lambda)/\left[1+\exp(\lambda)\right]$ is increasing in λ, the demand (the retailer's sales) under scenario 1 is greater than that in scenario 2.

We also use our results in Propositions 2 and 3 to calculate

$$p_1^* - p_2^* = -\frac{\lambda_1 - \lambda_2}{\beta} < 0,$$

which means that $p_1^* < p_2^*$. We also compute

$$i_1^* - i_2^* = \hat{\imath} \equiv \frac{(1-g)\left\{(1-g)\left[1+\exp(\lambda_1)\right]^2 - \left[1+\exp(\lambda_2)\right]^2\right\}}{\beta c} - g,$$

which may be positive or may be negative although $\lambda_1 > \lambda_2$. Similarly, we have

$$\begin{aligned}
\delta_1^* - \delta_2^* \quad = \quad \hat{\delta} \equiv & \frac{(1-g)(a-g)\exp(2\lambda_1) + \left[a(2-g) - g(3-2g)\right]\exp(\lambda_1) + g(g-2-\beta c) + a}{(1-g)^2\left[1+\exp(\lambda_1)\right]^2 - \beta cg} \\
& - \frac{(1-g)(a-g)\exp(2\lambda_2) + (1-g)\left[(2+g)a - 3g\right]\exp(\lambda_2)}{(1-g)^2\left[1+\exp(\lambda_2)\right]^2} \\
& - \frac{(2-a)g^2 - g\left[2+\beta c(1-a)\right] + a}{(1-g)^2\left[1+\exp(\lambda_2)\right]^2};
\end{aligned}$$

and we cannot determine the sign of $\delta_1^* - \delta_2^*$.

In addition,

$$\begin{aligned}
\pi_B^{1*} - \pi_B^{2*} \quad = \quad & -\frac{(1-a)\Theta}{\beta(1-g)\left[1+\exp(\lambda_2)\right]}, \\
& \times \exp(\lambda_2)\left[(1-g)\exp(2\lambda_2) - (g^2+g-2)\exp(\lambda_2) + \beta cg + 1 - g^2\right],
\end{aligned}$$

where

$$\begin{aligned}
\Theta \quad \equiv \quad & (1-g)\exp(\lambda_1)\left[1+\exp(\lambda_2)\right]\left[1+(1-g)\exp(\lambda_1)\right] \\
& - \exp(\lambda_2)\left[(1-g)\exp(2\lambda_2) - (g^2+g-2)\exp(\lambda_2) + \beta cg + 1 - g^2\right].
\end{aligned}$$

We also cannot determine the sign of $\pi_B^{1*} - \pi_B^{2*}$. Then, to compare the platform's profits under the two scenarios, we compute

$$\pi_E^{1*} - \pi_E^{2*} = \frac{a\Theta}{\beta\,(1-g)\,[1+\exp(\lambda_2)]},$$

which may be positive or may be negative.

This proposition is thus proved. ■

Proof of Proposition 5. For scenario 1, we compute the first-order derivative of system efficiency τ_1 w.r.t. g as

$$\frac{\partial \tau_1}{\partial g} = \frac{\exp(\lambda_1)}{\exp(\lambda_C)} \left\{ \frac{\partial \lambda_1}{\partial g} \left[2 - g + 2(1-g)\exp(\lambda_1)\right] - 1 - \exp(\lambda_1) \right\}.$$

Since

$$\frac{\partial \lambda_1}{\partial g} = \frac{1+\exp(\lambda_1)}{1+2\,(1-g)\exp(\lambda_1)},$$

we find that

$$\frac{\partial \tau_1}{\partial g} = \frac{\exp(\lambda_1)\,[1+\exp(\lambda_1)]}{\exp(\lambda_C)} \left[\frac{2(1-g)\exp(\lambda_1)+2-g}{2\,(1-g)\exp(\lambda_1)+1} - 1\right].$$

As $2(1-g)\exp(\lambda_1) + 2 - g > 2(1-g)\exp(\lambda_1) + 1$, we have $\partial\tau_1/\partial g > 0$, i.e., τ_1 is increasing in g.

For scenario 2, since $\partial\lambda_2/\partial g = -(\beta c)/\{[2\exp(2\lambda_2) + 3\exp(\lambda_2) + 1](1-g)^2\} < 0$, we have

$$\begin{aligned} \frac{\partial \tau_2}{\partial g} \quad = \quad & \frac{\beta c \exp(\lambda_2)}{(1-g)^2 \exp(\lambda_C)\,[1+\exp(\lambda_2)]^2} \left\{ \frac{\exp(2\lambda_2)+\exp(\lambda_2)}{2\exp(2\lambda_2)+3\exp(\lambda_2)+1} \right. \\ & \left. - \frac{\beta c g}{(1-g)\,[2\exp(2\lambda_2)+3\exp(\lambda_2)+1]} - 1 \right\}. \end{aligned}$$

Because $[\exp(2\lambda_2) + \exp(\lambda_2)]/[2\exp(2\lambda_2) + 3\exp(\lambda_2) + 1] < 1$, we find that $\partial\tau_2/\partial g < 0$, i.e., τ_2 is decreasing in g. Because the centralized payoff Π_C^* is independent on g, we have this proposition. ■