System-Wide Incentives to Trace Food Processing: A Cooperative-Game Analysis[a]

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Abstract

This paper uses cooperative game theory to analyze the incentives for firms to adopt product tracing in a three-tier food processing system with multiple farmers, one manufacturer, and one retailer. Firms that adopt the tracing system can either form a single coalition or create multiple coalitions, while non-adopters make decisions independently. Our analysis identifies equilibrium outcomes for all possible coalition structures, showing that collaboration between the manufacturer and retailer boosts system-wide efficiency, and fewer coalitions lead to greater overall benefits. We develop a coalition game in characteristic value form and prove that the game's core is always non-empty. The Center of Gravity of the Imputation Set-based value (CIS-value) is a core element of our game and it matches the nucleolus when the system includes at least three farmers. However, the CIS-value does not always ensure non-negative allocations for all coalition members. To resolve this, we introduce the Evenly-Split Value (ES-value), which stays within the core and guarantees positive allocations for every member of the tracing coalition.

Keywords: product tracing; food processing system; cooperative game; core; nucleolus; CIS-value.

1 Introduction

Food safety and traceability are paramount in the success of food processing systems, with around 64% of consumers preferring the brand that provides more in-depth product information (The Food Industry Association 2022). The traceability of products in a food processing system not only enhances food traceability for businesses but also provides greater product transparency for consumers (Gagliordi 2018). This, in turn, reduces the likelihood of deteriorated food (i.e., food that is no longer fresh but not yet inedible) reaching consumers (Gudmundsson 2023). Consequently, consumers feel more confident about food safety.

In practice, various technologies like RFID (Piramuthu, Farahani and Grunow 2013) and blockchain (Cui, Hu and Liu 2023) aid firms in tracing their products. In this paper, it is important to distinguish between "tracing" and "inspection." "Tracing" refers to the use of a specific technology or tool to identify the source of any product issue, whereas "inspection" involves the on-site examination of the product. For example, Walmart has actively pursued product tracing and encouraged its suppliers to trace food products using the IBM-developed blockchain-based tracking system (Pixelplex.io 2020). When firms trace their products, they can identify deteriorated products at their sites, preventing them from being further processed. This not only helps avoid costly food recalls or minimizes its expense for the firms (Saro 2024) but also results in a higher proportion of quality products (Moore 2024), potentially increasing consumer demand.

In practice, some firms in a food supply chain, particularly farmers, are often small to medium-sized enterprises (SMEs). These SMEs may face significant challenges in adopting a tracing system due to their limited financial resources and technological capabilities. Moore (2024) highlighted that the initial setup, operational, and compliance costs associated with implementing a tracing system can be prohibitively high for SMEs. Similarly, McEntire et al. (2010) demonstrated that SMEs often lack sufficient capital, labor, and technical expertise to implement such systems effectively.

Given these challenges, SMEs, such as farmers, may struggle to adopt a tracing system independently. However, system-wide cooperation within the food supply chain enables all members to share responsibilities for food product tracing. For this cooperation to succeed, it is crucial to ensure a "fair and acceptable" allocation of the system-wide profit gains resulting from the adoption of the tracing system. To address this need, cooperative game theory provides a robust methodology for determining fair allocation schemes. Accordingly, this paper utilizes cooperative game theory to analyze the incentives for all firms in the food supply chain to adopt a tracing system collectively.

We consider a three-tier food processing system consisting of multiple farmers, a single manufacturer, and a single retailer. In this system, the manufacturer purchases raw agricultural products from farmers to produce food items, which are then sold to the retailer for final sale to consumers. Firms have the option to form coalitions to implement product tracing and decide whether to adopt such systems based on their impact on profits. Firms not participating in tracing systems make independent pricing decisions, while coalition members collaborate to maximize their joint profits. Product tracing allows firms to monitor food quality throughout the supply chain, identifying and removing deteriorated products before they reach consumers. If deterioration occurs at a specific firm's site, that firm may face claims for compensation from downstream partners. Consumer demand depends on both the retail price and the percentage of deteriorated products sold. Any deteriorated products that reach consumers result in penalties for all members of the system, reflecting consumers' dissatisfaction.

Given the possibility of multiple coalitions forming within the system, we evaluate the decisions and profits of all firms and coalitions under various coalition structures. Our analysis reveals that cooperation between the manufacturer and retailer significantly enhances system efficiency, while system efficiency declines as the number of coalitions increases. We also formulate a transferable utility (TU) coalitional game in characteristic function form (see von Neumann and Morgenstern (1944, Ch. VI)) to study cooperation among supply chain members. In this framework, the characteristic value of a coalition is defined as the surplus achieved by its members under the worst-case scenario (see, e.g., Straffin (1993)), where non-coalition members refrain from adopting any tracing system. We demonstrate that the worst-case condition for any coalition arises when all members outside the coalition opt not to adopt any tracing system.

For this coalitional game, we analyze the core, the nucleolus, and the center of gravity of the imputation set-based value (CIS-value). Our findings demonstrate that the core of the game is always non-empty, even if the game is not superadditive or convex. The CIS-value consistently exists within the core and coincides with the nucleolus when there are at least three farmers. However, the CIS-value does not always guarantee a non-negative allocation to all coalition members when the system benefits from product tracing. To address this, we introduce the "evenly-split value" (ES-value), which evenly distributes the grand coalition's surplus among all firms. The ES-value resides within the core and ensures a positive allocation for all firms benefiting from system-wide product tracing.

We further find that imposing sufficiently high penalties for deteriorated products sold to consumers motivates the coalition involving the retailer to trace products at the retailer's site, removing all deteriorated products from the system. However, the coalition's decisions to trace at the manufacturer's or farmers' sites depend on penalty costs and the downstream partners' processing or production costs. Specifically, deteriorated products should be removed before reaching downstream partners if these costs are sufficiently high.

The paper is organized as follows: Section 2 reviews related literature. Section 3 introduces the models for the food processing system. Section 4 evaluates coalition profits, while Section 5 constructs and analyzes the coalitional game. Section 6 concludes with a summary of the main results.

2 Literature Review

This paper focuses on three key areas: food processing systems, product tracing, and transferable-utility (TU) coalitional games. To organize our literature review, we classify the most relevant studies into three categories: (i) game-theoretic analyses of product tracing, (ii) game-theoretic analyses of food processing systems, and (iii) TU coalitional games in supply chains.

2.1 Game-Theoretic Analyses of Product Tracing

In the realm of game-theoretic analyses related to product tracing, several notable studies have emerged: Saak (2016) explored a two-tier supply chain with two upstream firms and one downstream firm. His research demonstrated that perfect traceability of products may not always be the optimal choice, depending on specific conditions. Yao and Zhu (2020) studied the role of traceability in addressing product label misconduct. They discovered that organizations responsible for assigning product labels might incur higher costs by adopting tracing technology, potentially leading to unexpected consequences.

Recent studies have predominantly focused on the utilization of blockchain technology in product tracing: Fan et al. (2020) examined optimal pricing strategies in a three-tier supply chain, both with and without blockchain-based tracing technology. Their findings suggested that greater supply chain efficiency can be achieved through technology adoption, particularly when manufacturers bear the higher costs associated with it. Zhou et al. (2022) considered a two-tier supply chain where blockchain ensures reliable product information, thereby increasing transparency and demand. Their analysis highlighted the critical factors affecting supplier and retailer decisions on blockchain adoption. Liu et al. (2023) introduced consumer returns into the equation. They demonstrated that suppliers tend to favor high transparency levels when retailers absorb the return shipping cost.

In a dual-channel supply chain, Zhong et al. (2023) explored a dual-channel supply chain and discussed the potential benefits of government subsidies, either in quantity or innovation. Their research revealed that innovation subsidies could create a win-win-win situation for manufacturers, sales channels, and consumers.

Wu and Wang (2023) investigated the impact of blockchain traceability on the entire supply chain when customers purchase through an e-commerce platform. Their findings suggested that while the entire supply chain benefits from traceability, suppliers may face challenges. Consequently, platforms may consider offering subsidies to suppliers. Cui, Hu, and Liu (2023) analyzed the value of traceability-driven blockchain in various supply chain structures. In a three-tier serial supply chain, traceability consistently improved product quality and all firms' profits. Despite that, in a two-tier supply chain with two suppliers, traceability reduced product quality, adversely affecting suppliers but benefiting buyers, thereby creating incentive conflicts. These studies collectively provide insights into the complex dynamics of product tracing, especially in the context of emerging technologies like blockchain.

2.2 Game-Theoretic Analyses of Food Processing Systems

The literature surrounding food processing systems is a critical area of research due to its significance in ensuring food safety and meeting the needs of a growing global population. Several key studies and streams of research have contributed to our understanding. Borodin et al. (2016), Zhu et al. (2018) and Duan et al. (2020) have all provided comprehensive reviews of related literature. We review some papers that have developed game-theoretic models for a food processing systems analysis.

One approach involves developing principal-agent models for analyzing food processing systems: Dubois and Vukina (2004) utilized hog production data to estimate risk aversion among contract producers. Their findings indicated that increased risk aversion among growers could lead to reduced expected revenues. King et al. (2007) considered a multi-period problem, exploring the value of performance history in enhancing food safety. Pouliot and Sumner (2008) examined the relationship between traceability and liability for food safety lapses. They demonstrated how downstream firms can use traceability to shift liability upstream, reducing the likelihood of food safety issues.

Another approach focuses on sequential-move game models within food processing systems: Wang and Li (2012) introduced a pricing approach to minimize food spoilage waste and maximize the retailer's profit by dynamically identifying food shelf life. Wang et al. (2012) applied fuzzy set theory and an analytical hierarchy process to perform a structured analysis of aggregate food safety risk in a food supply chain. Piramuthu et al. (2013) explored liability allocation among supply chain members based on traceability accuracy in a perishable food supply chain. Chen et al. (2013) investigated ITC Limited's "e-Choupals" business model in India, analyzing ITC's incentives for providing this service and examining farmers' strategic quantity decisions. Tang et al. (2016) examined partially guaranteed pricing contracts in a food supply chain involving farmers and buyers. Wang and Chen

(2017) found that portfolio contracts could effectively coordinate fresh product supply chains. Dong et al. (2023) studied a three-tier food supply chain with multiple suppliers, assessing incentives for traceability technology adoption. They highlighted the dual effects of traceability, including saving uncontaminated food from disposal (pure traceability effect) and reducing purchase prices at each tier (strategic pricing effect). When the strategic pricing effect is significant, it may reduce the supplier's incentive to invest in contamination risk reduction, potentially leading to increased contamination risk. These studies collectively provide valuable insights into the complexities of food processing systems, including risk management, liability, coordination mechanisms, and the impact of emerging technologies like traceability.

2.3 TU Coalitional Games in Supply Chains

The utilization of transferable-utility (TU) coalitional games in supply chain management has been a rich area of research, focusing on various aspects of cooperation and coordination. These studies can be categorized into different areas, each addressing specific challenges in supply chain management:

(1) Cooperation among Retailers with Centralized Inventory Decisions: Some papers investigate cooperation among retailers who make centralized inventory decisions. These often revolve around optimizing inventory policies and allocation strategies. Examples include Montrucchio and Scarsini (2007) and Slikker et al. (2005).

(2) Cooperation with Transshipments among Retailers with Decentralized Inventory Decisions: Another category examines cooperation with transshipments among retailers who make decentralized inventory decisions. Here, the focus is on how retailers can collaborate effectively in a decentralized environment. Granot and Sošić (2003), Sošić (2006) are notable examples.

(3) Cooperation among Retailers in the Presence of Suppliers or Distributors: Some papers extend cooperation to include suppliers or distributors in the supply chain. These often explore how the entire supply chain network can collaborate to achieve common objectives. Chen and Chen (2013) and Zhang (2009) fall into this category.

Beyond inventory and supply chain coordination, TU games have been applied to various other supply chain areas, such as cost, revenue, and information sharing, cost savings through merging transportation demands, cooperation with fairness concerns, and coordination in closed-loop supply chains. Frisk et al. (2010), Leng and Parlar (2009), Lozano et al. (2013), Liu et al. (2021) , and Zheng et al. (2019) provide insights into different aspects of supply chain management and collaboration.

Nevertheless, the applications of cooperative game theory to the specific problem of product tracing in food processing systems has been relatively limited. Given the growing importance of traceability technologies in the food industry, there is a need for research in this area. Notably, major players like Walmart, Tyson Foods, and Nestlé have begun adopting tracing technologies, highlighting the timeliness and relevance of research in this domain.

Our research on vertical cooperation between firms at different tiers in a three-tier food processing system addresses an important and emerging issue in the field of supply chain management. By developing a TU coalitional game framework for this problem, our work contributes to the understanding of how firms can collaborate effectively in the context of product tracing and food safety in the modern food processing industry.

3 The Food Processing Model

In a three-tier food processing system, n farmers F_k $(k = 1, \ldots, n)$ produce complementary agricultural products incurring unit costs c_{F_k} $(k = 1, \ldots, n)$ and sell them to a manufacturer at prices p_{F_k} $(k = 1, \ldots, n)$. The manufacturer processes these into food products at unit cost c_M, then sells them to a retailer at price p_M. The retailer incurs unit processing cost c_R and sells the final products to consumers at unit price p_R. Along the supply chain, products may deteriorate due to factors like improper processing, storage, or transportation, with deterioration categorized into two types:

- Deterioration during sojourn. Products initially of high quality deteriorate while at a firm's site. The proportion of such products at a firm i is $1 - \lambda_i$. For example, $1 - \lambda_{F_k}$ represents the deterioration rate at farmer F_k's site.
- Pre-existing deterioration. Products arrive at a firm's site already deteriorated. Identifying and removing these products depends on whether the firm adopts a tracing technology.

Deterioration can occur at any stage of the supply chain, and removing deteriorated products is critical to maintaining quality and avoiding penalties. Tracing systems, such as blockchain technology, enable firms to monitor the status of products and ensure quality. According to the International Trade Center (2015), traceability systems have become essential in commercial negotiations and product specifications. For example, Walmart and manufacturers like Nestlé and Tyson Foods have implemented blockchain-based tracing systems to improve food quality (Pixelplex.io 2020, Jones 2020, Shoup 2018). These systems are increasingly incorporated into sustainability criteria for procurement, creating incentives for upstream firms, such as farmers, to adopt traceability.

Firms in the supply chain decide whether to adopt a tracing system, denoted by $y_i \in \{0, 1\}$ for $i \in \Gamma = \{F_1, \ldots, F_n, M, R\}$), where $y_i = 1$ indicates adoption. Tracing incurs a unit cost t_i. If a firm does not trace $(y_i = 0)$, it sells all deteriorated products to its downstream partner without identifying their condition. However, if tracing is adopted ($y_i = 1$), the firm identifies and removes all deteriorated products at its site.

Adopting a tracing system allows firms to identify the source of deteriorated products , enabling compensation claims along the supply chain. We denote the unit compensation for these claims by $\boldsymbol{\gamma}$. For instance, if a manufacturer traces deterioration to a farmer F_k the farmer pays the manufacturer a per-unit compensation $\gamma_{F_k}^M$. Similarly, the retailer may charge the manufacturer γ_M^R or a farmer $\gamma_{F_k}^R$ for traced deterioration. Compensation decisions $\{\gamma_{F_k}^M\}_{k=1}^n$, γ_M^R and $\{\gamma_{F_k}^R\}_{k=1}^n$ are decision variables for the manufacturer and retailer. This mechanism is exemplified by Walmart's recommendation for its suppliers to adopt tracing systems.

Firms that do not trace products cannot claim compensation and must jointly absorb penalties when consumers identify deteriorated products. Penalties are distributed across the supply chain, with farmers ($\gamma_{F_k}^0$), the manufacturer (γ_M^0), and the retailer ($\gamma_{R,}^0$) incurring respective costs. The total penalty cost for the entire supply chain is $\gamma_\Gamma^0 \equiv \sum_{i\in\Gamma} \gamma_i^0$. Tracing systems and cooperative coalitions in the supply chain are crucial to mitigating these penalties and maintaining quality, while strategic decisions about tracing adoption depend on balancing costs, responsibilities, and business relationships.

In summary, the sequence of activities in the food processing system unfolds as follows:

1. Coalitional Game and Tracing System Adoption: All members decide whether to adopt a tracing system by playing a coalitional game. They may form multiple coalitions for adoption, each incurring a fixed cost proportional to its size (i.e., $|S_k|\Delta$, where $|S_k|$ is the number of firms in coalition S_k).
2. Tracing Decisions: Coalitions make simultaneous tracing decisions ($y_i \in \{0,1\}$) for each firm ($i \in \Gamma$). These decisions determine the proportion of deteriorated products sold to consumers. Firms not adopting a tracing system default to $y = 0$.
3. Compensation Decisions: The manufacturer and retailer determine compensations ($\{\gamma_{F_k}^M\}_{k=1}^n$ and $\{\gamma_M^R, \{\gamma_{F_k}^R\}_{k=1}^n\}$) for deteriorated products identified through tracing. Firms that do not adopt tracing cannot claim compensation.
4. Farmers' Pricing Decisions: Farmers set their prices ($\{p_{F_k}\}_{k=1}^n$) for agricultural products sold to the manufacturer.
5. Manufacturer's Wholesale Pricing: The manufacturer determines the wholesale price (p_M) for products sold to the retailer.
6. Retailer's Pricing: The retailer sets the final retail price (p_R) for consumers.

Consumer demand depends on both the retail price (p_R) and the proportion of deteriorated products among those sold to consumers (α). Building on the model proposed by Huang et al. (2013), the demand function is expressed in multiplicative form as:

$$\omega(p_R, \alpha) = (A - hp_R)D(\alpha), \tag{1}$$

where $A - hp_R$ represents the demand multiplier influenced by the retail price, with $A/h > \Sigma_{i\in\Gamma} c_i + (1 - \Pi_{i\in\Gamma}\lambda_i)\gamma_\Gamma^0$. This condition ensures profitability when no firms adopt a tracing system. $D(\alpha)$ ($\alpha \in [0, 1]$) denotes the demand multiplier dependent on the proportion of deteriorated products. It satisfies two key properties:

- $D(1) = 0$, reflecting that if all products are deteriorated, demand becomes zero because rational consumers will not purchase them.
- $D'(\alpha) < 0$, indicating that as the proportion of deteriorated products increases, consumer demand decreases.

This multiplicative form, particularly the linear structure of the price-dependent multiplier, is widely used in game-theoretic analyses of food processing systems (e.g., Chen et al., (2013)), making it a practical and insightful approach for modeling consumer demand behavior in such contexts.

Let $\beta = 1 - \alpha$ measure the *quality of the product*, and compute the *quality elasticity of demand* as:

$$E(\beta) \equiv \frac{\partial D(1-\beta)}{\partial \beta}\frac{\beta}{D(1-\beta)} = -\frac{\beta D'(1-\beta)}{D(1-\beta)} \geq 0.$$

With $\beta \geq \Pi_{i\in\Gamma}\lambda_i$, we define

$$E_{\max} \equiv \max_{\Pi_{i\in\Gamma}\lambda_i \leq \beta \leq 1} E(\beta) \text{ and } E_{\min} \equiv \min_{\Pi_{i\in\Gamma}\lambda_i \leq \beta \leq 1} E(\beta)$$

as the maximum and minimum quality elasticities, respectively. Using this framework, we calculate the profits for each firm in the supply chain in the following.

Denote by $\alpha_S \in [0, 1]$ the proportion of deteriorated products that are sold to consumers if the members of S ($S \subseteq \Gamma$) adopt the tracing system. To illustrate the calculation of α_S, we compute α_Γ and the corresponding expected profits for the firms in the food processing system. As all firms adopt the tracing system and the coalitions determine their optimal tracing decisions y_i ($i \in \Gamma$), some of the deteriorated products can be identified and removed. When farmer F_k ($k = 1, \ldots, n$) produces Q_{F_k} products at cost $c_{F_k}Q_{F_k}$, $(1-\lambda_{F_k})Q_{F_k}$ products deteriorate at the farm. As farmer F_k's tracing decision is y_{F_k}, he can identify $(1 - \lambda_{F_k})y_{F_k}Q_{F_k}$ deteriorated products at cost $t_{F_k}y_{F_k}Q_{F_k}$ and remove them from the system. As a result, farmer F_k sells to the manufacturer $[1 - y_{F_k}(1 - \lambda_{F_k})]Q_{F_k}$

products, among which $\lambda_{F_k} Q_{F_k}$ products are good and $(1-y_{F_k})(1-\lambda_{F_k})Q_{F_k}$ products are defective. That is, the fraction of farmer F_k's products sold to the manufacturer that are deteriorated is given by $(1-y_{F_k})(1-\lambda_{F_k})/[1-y_{F_k}(1-\lambda_{F_k})]$.

If the manufacturer orders Q_M products from each farmer, then we find $Q_M = [1-y_{F_k}(1-\lambda_{F_k})]Q_{F_k}$, for $k=1,\ldots,n$. Since the manufacturer produces Q_M food products and traces $y_M Q_M$ of them, the number of farmer F_k's deteriorated agricultural products that are identified by the manufacturer is $\frac{(1-y_{F_k})(1-\lambda_{F_k})}{1-y_{F_k}(1-\lambda_{F_k})} y_M Q_M$. Thus, farmer F_k pays to the manufacturer the compensation $\frac{(1-y_{F_k})(1-\lambda_{F_k})}{1-y_{F_k}(1-\lambda_{F_k})} y_M Q_M \gamma^M_{F_k}$. Farmer F_k's remaining deteriorated food products in amount of $\frac{(1-y_{F_k})(1-\lambda_{F_k})(1-y_M)}{1-y_{F_k}(1-\lambda_{F_k})} Q_M$ are sold to the retailer, in which $\frac{(1-y_{F_k})(1-\lambda_{F_k})(1-y_M)}{1-y_{F_k}(1-\lambda_{F_k})} y_R Q_M$ are identified by the retailer. Thus, farmer F_k pays to the retailer the compensation $\frac{(1-y_{F_k})(1-\lambda_{F_k})(1-y_M)}{1-y_{F_k}(1-\lambda_{F_k})} y_R Q_M \gamma^R_{F_k}$.

Then, we calculate the number of deteriorated products that are sold to the consumers. The manufacturer produces Q_M food products and traces y_M of them. It is evident that a food product is of good quality, if and only if all the complementary food products used in its production are in good condition and no deterioration occurs at the manufacturer's site. Therefore, the fraction of food products with good quality can be calculated as $\lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$. As a consequence, the manufacturer identifies and removes

$$y_M Q_M \left[1 - \lambda_M \prod\nolimits_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]$$

defective products at his site; and, the number of products that move to the retailer is

$$Q_R = \left[1 - y_M + y_M \lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] Q_M,$$

in which the number of good quality products is $\lambda_M Q_M \prod\limits_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$. As the deterioration at the retailer's site occurs with fraction $1-\lambda_R$ and the retailer traces y_R of the received products, the number of deteriorated products that are sold to consumers is

$$\begin{aligned} & (1-y_R)\left[(1-y_m)\left(1-\lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right) + (1-\lambda_R)\lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] Q_M \\ = \;& (1-y_R)\left[1 - y_m + (y_m - \lambda_R)\lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] Q_M. \end{aligned}$$

As a consequence, farmer F_k's expected profit is

$$\pi_{F_k}(\mathbf{p}, \boldsymbol{\gamma}; \mathbf{y}) = [p_{F_k} - \phi_{F_k}(\mathbf{y})] Q_M, \tag{2}$$

where $\mathbf{p} \equiv (p_i)_{i\in\Gamma}$, $\boldsymbol{\gamma} \equiv ((\gamma^M_{F_k})^n_{k=1}, \gamma^R_M, (\gamma^R_{F_k})^n_{k=1})$, $\mathbf{y} \equiv (y_i)_{i\in\Gamma}$, and

$$\begin{aligned}
\phi_{F_k}(\boldsymbol{\gamma};\mathbf{y}) \quad \equiv \quad & \frac{(1-y_{F_k})(1-\lambda_{F_k})y_M\gamma^M_{F_k} + c_{F_k} + t_{F_k}y_{F_k} + (1-y_{F_k})(1-\lambda_{F_k})(1-y_M)y_R\gamma^R_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})} \\
& + \gamma^0_{F_k}(1-y_R)\left[1-y_m+(y_m-\lambda_R)\lambda_M\prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right].
\end{aligned}$$

After the manufacturer purchases and processes all the food products, he produces Q_M products at cost $(c_M + \sum_{k=1}^n p_{F_k})Q_M$ and traces y_M of Q_M products at cost $t_M y_M Q_M$. The fraction of food products with good quality can be calculated as $\lambda_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$. As a result, the manufacturer identifies and removes

$$y_M Q_M \left[1-\lambda_M \prod\nolimits_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]$$

defective products at his site; and, the number of products that move to the retailer is

$$Q_R = \left[1-y_M+y_M\lambda_M\prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]Q_M,$$

in which the number of good quality products is $\lambda_M Q_M \prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$. Note that $(1-\lambda_M)Q_M$ products are deteriorated at the manufacturer's site, and $y_M(1-\lambda_M)Q_M$ are removed by the manufacturer. Therefore, the retailer receives $(1-y_M)(1-\lambda_M)Q_M$ products that are deteriorated at the manufacturer's site. Since the retailer traces y_R of food products, the manufacturer pays to the retailer the compensation $y_R Q_M \gamma^R_M(1-y_M)(1-\lambda_M)$. Thereby, the manufacturer's expected profit is

$$\pi_M(\mathbf{p},\boldsymbol{\gamma};\mathbf{y}) = p_M Q_M\left[1-y_M+y_M\lambda_M\prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] - Q_M\sum_{k=1}^n p_{F_k} - Q_M\phi_M(\mathbf{y}), \qquad (3)$$

where

$$\begin{aligned}
\phi_M(\boldsymbol{\gamma};\mathbf{y}) \quad \equiv \quad & c_M + y_M t_M + y_R\gamma^R_M(1-y_M)(1-\lambda_M) - y_M\sum_{k=1}^n\left[\frac{(1-y_{F_k})(1-\lambda_{F_k})}{1-y_{F_k}(1-\lambda_{F_k})}\gamma^M_{F_k}\right] \\
& + \gamma^0_M(1-y_R)\left[1-y_m+(y_m-\lambda_R)\lambda_M\prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right].
\end{aligned}$$

After the retailer receives Q_R products from the manufacturer, his tracing decision is y_R and thus he incurs total cost $(c_R + p_M + y_R t_R)Q_R$. As the deterioration at the retailer's site occurs with fraction $1-\lambda_R$, the number of high-quality products that deteriorate at the retailer's site is

$$Q_M(1-\lambda_R)\lambda_M\left[\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right].$$

Therefore, we can calculate the total number of deteriorated products at the retailer's site as

$$\begin{aligned}&(1-y_M)\left[1-\lambda_M\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]Q_M+(1-\lambda_R)\lambda_M\left[\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]Q_M\\ =\;&\left[1-y_M+(y_M-\lambda_R)\lambda_M\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]Q_M.\end{aligned}$$

The retailer can identify and remove defective products if $y_R=1$. Hence, the total number of products that the retailer sells to consumers is

$$Q\equiv Q_R-y_RQ_M\left[1-y_M+(y_M-\lambda_R)\lambda_M\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]=Q_M\varphi(\mathbf{y}), \tag{4}$$

in which

$$\varphi(\mathbf{y})\equiv(1-y_R)\left[1-y_M+(y_M-\lambda_R)\lambda_M\prod\nolimits_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]+\lambda_R\lambda_M\prod\nolimits_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}.$$

Among the sold products, $\lambda_R\lambda_MQ_M\{\prod_{k=1}^{n}\lambda_{F_k}/[1-y_{F_k}(1-\lambda_{F_k})]\}$ products are of good quality. Using the above, we compute

$$\alpha_\Gamma=\frac{\varphi(\mathbf{y})-\lambda_R\lambda_M\prod_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}}{\varphi(\mathbf{y})}=1-\frac{\lambda_R\lambda_M\prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1-y_{F_k}(1-\lambda_{F_k})]},$$

and obtain the retailer's expected profit as

$$\begin{aligned}\pi_R(\mathbf{p},\boldsymbol{\gamma};\mathbf{y})\;=\;&p_RQ_M\left\{(1-y_R)(1-y_M)+(y_M-y_Ry_M+y_R\lambda_R)\lambda_M\prod\nolimits_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right\}\\&-p_MQ_M\left[1-y_M+y_M\lambda_M\prod\nolimits_{k=1}^{n}\frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right]-Q_M\phi_R(\boldsymbol{\gamma};\mathbf{y}),\end{aligned} \tag{5}$$

where

$$
\begin{aligned}
\phi_R(\boldsymbol{\gamma};\mathbf{y}) \;\equiv\;& (c_R + y_R t_R)\left[1 - y_M + y_M\lambda_M \prod\nolimits_{k=1}^{n} \frac{\lambda_{F_k}}{1 - y_{F_k}(1-\lambda_{F_k})}\right] \\
& -\sum_{k=1}^{n} \frac{(1-y_{F_k})(1-\lambda_{F_k})(1-y_M)}{1-y_{F_k}(1-\lambda_{F_k})} y_R \gamma_{F_k}^R - (1-y_M)(1-\lambda_M) y_R \gamma_M^R \\
& +\gamma_R^0 (1-y_R)\left[1 - y_m + (y_m - \lambda_R)\lambda_M \prod_{k=1}^{n} \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right].
\end{aligned}
$$

For a set $S \subset \Gamma$, the value of α_S can be obtained by setting $y_i = 0$ ($i \in \Gamma\backslash S$) in α_Γ, as a product is traceable at firm i if that firm attempts to trace the products. When all members in set S adopt the tracing system, we also calculate the expected profits of the firms in the food processing system by letting $y_i = 0$ ($i \in \Gamma\backslash S$) in (2), (3), and (5).

It follows from (4) and (1) that

$$
\varphi(\mathbf{y})Q_M = (A - hp_R)D(\alpha) = (A - hp_R) \times D\left(1 - \frac{\lambda_R\lambda_M \prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1 - y_{F_k}(1-\lambda_{F_k})]}\right). \tag{6}
$$

Using (6), we can rewrite (2), (3), and (5) as follows:

$$
\begin{aligned}
\pi_{F_j}(\mathbf{p},\boldsymbol{\gamma};\mathbf{y}) \;=\;& \frac{(A-hp_R)(p_{F_j} - \phi_{F_j}(\boldsymbol{\gamma};\mathbf{y}))}{\varphi(\mathbf{y})} \times D\left(1 - \frac{\lambda_R\lambda_M \prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1 - y_{F_k}(1-\lambda_{F_k})]}\right), \\
\pi_M(\mathbf{p},\boldsymbol{\gamma};\mathbf{y}) \;=\;& \frac{(A-hp_R)}{\varphi(\mathbf{y})} \times D\left(1 - \frac{\lambda_R\lambda_M \prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1 - y_{F_k}(1-\lambda_{F_k})]}\right) \\
& \times \left\{ p_M\left[1 - y_M + y_M\lambda_M \prod\nolimits_{k=1}^{n} \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] - \sum\nolimits_{k=1}^{n} p_{F_k} - \phi_M(\boldsymbol{\gamma};\mathbf{y})\right\}, \\
\pi_R(\mathbf{p},\boldsymbol{\gamma};\mathbf{y}) \;=\;& (A-hp_R) \times D\left(1 - \frac{\lambda_R\lambda_M \prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1 - y_{F_k}(1-\lambda_{F_k})]}\right) \\
& \times \left\{ p_R - \frac{p_M\left[1 - y_M + y_M\lambda_M \prod_{k=1}^{n} \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}\right] + \phi_R(\boldsymbol{\gamma};\mathbf{y})}{\varphi(\mathbf{y})}\right\}.
\end{aligned}
$$

The total profit for the food processing system is

$$
\pi(p_r;\mathbf{y}) = (A - hp_R) \times \left(p_R - \frac{\phi(\mathbf{y})}{\varphi(\mathbf{y})}\right) \times D\left(1 - \frac{\lambda_R\lambda_M \prod_{k=1}^{n}\lambda_{F_k}}{\varphi(\mathbf{y})\prod_{k=1}^{n}[1 - y_{F_k}(1-\lambda_{F_k})]}\right). \tag{7}
$$

where $\phi(\mathbf{y}) \equiv \sum_{i\in\Gamma}\phi_i(\boldsymbol{\gamma};\mathbf{y})$, in which compensation decisions $\boldsymbol{\gamma} \equiv ((\gamma_{F_k}^M)_{k=1}^n, \gamma_M^R, (\gamma_{F_k}^R)_{k=1}^n)$ are canceled in the sum.

4 Optimal Decisions and Profits for Different Coalitions

We derive the optimal decisions and profits for individual firms and coalitions within the food processing system. The analysis begins with a benchmark scenario where no firm implements product tracing. We then explore all possible product tracing coalitions within the system.

4.1 The Benchmark Case: No Product Tracing

We calculate the profits for all firms in the benchmark scenario where no firm implements product tracing (i.e., $y_i = 0$, $\forall i \in \Gamma$).In this case, no deteriorated products are identified, resulting in all such products being processed and reaching consumers. Let

$$\Psi(\mathbf{y}) \equiv \left[\max\left(0, \frac{A}{h} - \frac{\phi(\mathbf{y})}{\varphi(\mathbf{y})}\right)\right]^2 \times D\left(1 - \frac{\lambda_R \lambda_M \prod_{k=1}^n \lambda_{F_k}}{\varphi(\mathbf{y}) \prod_{k=1}^n [1 - y_{F_k}(1-\lambda_{F_k})]}\right). \tag{8}$$

Then, the maximum system-wide profit for this case is $h\Psi(\mathbf{0})/4$, which is obtained at $\bar{p}_R^{\emptyset} \equiv \frac{A}{2h} + \frac{\phi(\mathbf{0})}{2\varphi(\mathbf{0})} = \frac{A}{2h} + \frac{\phi(\mathbf{0})}{2}$.

For the decentralized case, we determine the Stackelberg equilibrium and calculate the resulting profits when no firm implements product tracing. In this sequential pricing framework, the farmers first simultaneously decide their prices, p_{F_k} $(k = 1, \ldots, n)$. The manufacturer then observes these decisions and sets its price, p_M, which is followed by the retailer determining the final retail price, p_R, based on the manufacturer's choice.

From (7), it follows that for the empty coalition, the retail price satisfies $A/h > p_R > \phi(\mathbf{0})/\varphi(\mathbf{0}) = \phi(\mathbf{0})$, and $A/h - \phi(\mathbf{0})$ serves as a measure of the profitability of the entire system.

Proposition 1 *When no firm traces the products (i.e., $S = \emptyset$, where S denotes the set of firms that adopt the tracing system), the equilibrium prices are determined as follows:*

$$\begin{cases} p_R^{\emptyset} = \dfrac{4n+3}{4n+4}\dfrac{A}{h} + \dfrac{1}{4n+4}\left[\sum\limits_{i\in\Gamma} c_i + \left(1 - \prod\limits_{i\in\Gamma}\lambda_i\right)\gamma_\Gamma^0\right], \\ p_M^{\emptyset} = \dfrac{\dfrac{(2n+1)A}{h} + \sum\limits_{i\in\Gamma} c_i + \left(1 - \prod\limits_{i\in\Gamma}\lambda_i\right)\gamma_\Gamma^0}{2(n+1)} - c_R - \left(1 - \prod\limits_{i\in\Gamma}\lambda_i\right)\gamma_R^0, \\ p_{F_k}^{\emptyset} = \dfrac{\frac{A}{h} - \sum\limits_{i\in\Gamma} c_i - \left(1 - \prod\limits_{i\in\Gamma}\lambda_i\right)\gamma_\Gamma^0}{n+1} + c_{F_k} + \left(1 - \prod\limits_{i\in\Gamma}\lambda_i\right)\gamma_{F_k}^0, \textit{ for } k = 1, ..., n. \end{cases} \tag{9}$$

The resulting profits for the firms are

$$\pi_{F_k}^{\emptyset} = \frac{h\Psi(\mathbf{0})}{4(n+1)^2},\ \textit{for } k = 1, \ldots, n;\ \pi_M^{\emptyset} = \frac{h\Psi(\mathbf{0})}{8(n+1)^2};\ \textit{and } \pi_R^{\emptyset} = \frac{h\Psi(\mathbf{0})}{16(n+1)^2}.$$

Proof. When no firm traces products, in the sixth stage, the retailer maximizes $\pi_R(\mathbf{p}, \boldsymbol{\gamma}; \mathbf{0})$ to find the optimal retail price as

$$p_R = \frac{p_M + c_R + \left(1 - \prod_{i\in\Gamma} \lambda_i\right)\gamma_R^0}{2} + \frac{A}{2h}.$$

In the fifth stage, substituting the above p_R into $\pi_M(\mathbf{p}, \boldsymbol{\gamma}; \mathbf{0})$, the manufacturer's optimal wholesale price is

$$p_M = \frac{\sum_{k=1}^{n} p_{F_k} + c_M + \left(1 - \prod_{i\in\Gamma} \lambda_i\right)(\gamma_M^0 - \gamma_R^0) - c_R}{2} + \frac{A}{2h}.$$

In the fourth stage, substituting the above p_R and p_M into $\pi_{F_k}(\mathbf{p}, \boldsymbol{\gamma}; \mathbf{0})$ for $k = 1, ..., n$, the Nash equilibrium of the farmers satisfies

$$p_{F_k} - c_{F_k} - \left(1 - \prod_{i\in\Gamma} \lambda_i\right)\gamma_{F_k}^0 = \frac{A}{h} - \sum_{j=1}^{n} p_{F_j} - c_M - \left(1 - \prod_{i\in\Gamma} \lambda_i\right)(\gamma_M^0 + \gamma_R^0) - c_R.$$

Therefore, we find the equilibrium decisions and the profits as shown in Proposition 1. ■

Proposition 1 demonstrates that in the decentralized scenario, the total profit is given by $\sum_{i\in\Gamma} \pi_i^{\emptyset} = (4n+3)h\Psi(\mathbf{0})/[16(n+1)^2]$. Comparing this to the maximum system-wide profit of $h\Psi(\mathbf{0})/4$, we observe an efficiency loss of $(2n+1)^2/(2n+2)^2$ resulting from decentralized pricing decisions.

Let $S \subseteq \Gamma$ denote the set of all members adopting the tracing system. These members in S may form $\tau \geq 1$ coalitions, which we represent as the coalition structure $\Theta = \{S_1, \ldots, S_\tau\}$, where $S = \cup_{k=1}^{\tau} S_k$ and $S_j \cap S_k = \emptyset$. We proceed to analyze five possible coalition structures: (i) $S \cap \{M, R\} = \emptyset$, (ii) $S \cap \{M, R\} = \{M\}$, (iii) $S \cap \{M, R\} = \{R\}$, (iv) $\{M, R\} \subset S$, M and R are in different coalitions, and (v) $\{M, R\} \subset S$, M and R are in the same coalition.

4.2 The Coalition Structure When Neither the Manufacturer Nor the Retailer Traces the Products ($S \cap \{M, R\} = \emptyset$)

We derive the equilibrium decisions for coalition structure Θ when the manufacturer and the retailer do not adopt the tracing system. A farmer F_k ($k \in \{1, \ldots n\}$) who does not trace products incurs a total production cost of $c_{F_k} Q_M$, where Q_M represents the quantity ordered by the manufacturer.

However, when farmer F_k adopts tracing, he needs to produce Q_M/λ_{F_k} units to ensure that Q_M units meet the required quality standards after accounting for deterioration. The total cost for a tracing farmer becomes $(c_{F_k}+t_{F_k})Q_M/\lambda_{F_k}$, where t_{F_k} represents the tracing cost per unit. Hence, we can calculate the *unit identification cost* of F_k as

$$T_{F_k} \equiv \frac{(c_{F_k}+t_{F_k})Q_M/\lambda_{F_k} - c_{F_k}Q_M}{Q_M} = \frac{(1-\lambda_{F_k})c_{F_k}}{\lambda_{F_k}} + \frac{t_{F_k}}{\lambda_{F_k}}, \text{ for } k=1,\ldots,n; \text{ and } T_\Theta \equiv \sum_{i\in S} T_i.$$

where T_Θ denotes the unit identification cost jointly incurred by all firms in coalition S under structure Θ.

Given that the coalition structure Θ allows for $2^{|S|}$ possible tracing decisions, directly comparing all possible scenarios to identify the optimal tracing configuration would be computationally infeasible. However, we derive sufficient conditions to determine the equilibrium tracing decision ($y_j^\Theta = 1$ or $y_j^\Theta = 0$) for each member S_l ($l \in \{1,\ldots,\tau\}$).

Proposition 2 *In a coalition structure $\Theta = \{S_1, ..., S_\tau\}$ when only the farmers may trace products ($S = \cup_{k=1}^{\tau} S_k \subseteq \{F_1,\ldots,F_n\}$), we find the equilibrium tracing decisions as $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\}, i\in S} \Psi(\mathbf{y})$ ($y_i = 0$ for $i \in \Gamma\backslash S$). Moreover,*

1. *farmer F_j ($j \in S$) traces all the products (i.e., $y_j^\Theta = 1$) if*

$$\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta\right]^+ \geq \frac{2}{E_{\min}}\frac{T_j}{1-\lambda_j} - \frac{2}{E_{\min}}\left(\prod_{i\in\Gamma}\lambda_i\right)\gamma_\Gamma^0.$$

2. *farmer F_j ($j \in S$) does not trace products (i.e., $y_j^\Theta = 0$) if*

$$\frac{A}{h} - \phi(\mathbf{0}) \leq \frac{2}{E_{\max}}\frac{\lambda_j T_j}{1-\lambda_j} - \left[\left(1+\frac{2}{E_{\max}}\right)\prod_{i\in\Gamma\backslash S}\lambda_i - \prod_{i\in\Gamma}\lambda_i\right]\gamma_\Gamma^0.$$

The corresponding profits of the coalitions and other firms are

$$\begin{aligned}
\pi_{S_l}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2}, \text{ for } l \in \{1,...,\tau\};\\
\pi_i^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2}, \text{ for } i \in (\Gamma\backslash\{R,M\})\backslash S;\\
\pi_M^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{8(n-|S|+\tau+1)^2}; \text{ and } \pi_R = \frac{h\Psi(\mathbf{y}^\Theta)}{16(n-|S|+\tau+1)^2},
\end{aligned}$$

where $\Psi(\mathbf{y})$ is defined as in (8).

Proof. Since the manufacturer and the retailer do not adopt the tracing system, we do not need to consider decisions $\boldsymbol{\gamma} \equiv (\gamma_i)_{i\in\Gamma\backslash\{R\}}$ and $y_M = y_R = 0$. Given $\mathbf{y} = (y_i)_{i\in\Gamma}$, in which $y_i = 0$ if $i \in \Gamma\backslash S$, then the farmers in the same coalition S_k ($k \in \{1,\ldots,\tau\}$) determine a single price $p_{S_k} = \sum_{i\in S_k} p_i$ to maximize the coalition's total profit. In this proof, $\phi_R(\boldsymbol{\gamma};\mathbf{y})$, $\phi_M(\boldsymbol{\gamma};\mathbf{y})$, and $\phi_{F_j}(\boldsymbol{\gamma};\mathbf{y})$ ($j \in \{1,\ldots,n\}$) are independent of $\boldsymbol{\gamma}$ (i.e., no decision is made at stage three), and $\varphi(\mathbf{y}) = 1$ when $y_M = y_R = 0$. The calculations of the equilibrium decisions from stage six to stage four are very similar to the proof of Proposition 1. Thus we omit the detailed calculations and list the profits of all coalitions and members that do not adopt the tracing system under given $\mathbf{y}$ as

$$\begin{aligned}
\pi_{S_l} &= \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2}, \text{ for } l \in \{1,...,\tau\};\\
\pi_i &= \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2}, \text{ for } i \in (\Gamma\backslash\{R,M\})\backslash S;\\
\pi_M &= \frac{h\Psi(\mathbf{y})}{8(n-|S|+\tau+1)^2}; \text{ and } \pi_R = \frac{h\Psi(\mathbf{y})}{16(n-|S|+\tau+1)^2},
\end{aligned}$$

in which π_{S_l} is the profit of coalition S_l.

In the second stage, all coalitions maximize the same objective $\Psi(\mathbf{y})$, the products tracing decisions satisfy $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\},i\in S} \Psi(\mathbf{y})$ ($y_i = 0$ for $i \in \Gamma\backslash S$) and we can write the profits of the coalition and all firms as in this proposition. Letting $z_i \equiv 1/(1-y_i+y_i\lambda_i)$ for $i \in S$, the coalition solves the following problem:

$$\begin{aligned}
\max_{\substack{z_i\in\{1,1/\lambda_i\}\\ i\in S}} \quad & B_S \equiv \left(\tfrac{A}{h} - \zeta\gamma_\Gamma^0 - \sum_{i\in S}\left(c_i + \tfrac{t_i}{1-\lambda_i}\right)z_i - \sum_{i\in\Gamma\backslash S} c_i + \sum_{i\in S}\tfrac{t_i}{1-\lambda_i}\right)^2 D(\zeta)\\
\text{s.t.} \quad & \frac{A}{h} \geq \zeta\gamma_\Gamma^0 + \sum_{i\in S}\left(c_i + \tfrac{t_i}{1-\lambda_i}\right)z_i + \sum_{i\in\Gamma\backslash S} c_i - \sum_{i\in S}\tfrac{t_i}{1-\lambda_i},\ \zeta = 1 - \prod_{i\in\Gamma}\lambda_i\prod_{i\in S} z_i.
\end{aligned}$$

Given $j \in S$, we find

$$\frac{z_j\partial B_S}{B_S\partial z_j} = \frac{2\left[\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0 - c_jz_j - \frac{t_j}{1-\lambda_j}z_j\right]}{\frac{A}{h} + \sum_{i\in S}\frac{t_i}{1-\lambda_i} - \left(1-\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0 - \sum_{i\in S}\left(c_i+\frac{t_i}{1-\lambda_i}\right)z_i - \sum_{i\in\Gamma\backslash S}c_i} + E\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right).$$

(1) Conditions for $y_j^\Theta = 0$. When

$$\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta\right]^+ \geq \frac{2}{E_{\min}}\frac{T_j}{1-\lambda_j} - \frac{2}{E_{\min}}\left(\prod_{i\in\Gamma}\lambda_i\right)\gamma_\Gamma^0,$$

we find

$$\frac{E\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)}{2}\left[\frac{A}{h}-\sum_{i\in\Gamma}c_i-\left(1-\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0-\sum_{i\in S}\frac{\lambda_i(z_i-1)T_i}{1-\lambda_i}\right]$$
$$\geq\ \frac{\lambda_j z_j T_j}{1-\lambda_j}-\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0,$$

which means $\partial B_S/\partial z_j \geq 0$, and thus $y_j^\Theta = 0$.

(2) Conditions for $y_j^\Theta = 1$. When

$$\frac{A}{h}-\phi(\mathbf{0})-\left(\prod_{i\in\Gamma}\lambda_i-\prod_{i\in\Gamma\backslash S}\lambda_i\right)\gamma_\Gamma^0\leq\frac{2}{E_{\max}}\frac{\lambda_j T_j}{1-\lambda_j}-\frac{2}{E_{\max}}\left(\prod_{i\in\Gamma\backslash S}\lambda_i\right)\gamma_\Gamma^0,$$

we find

$$\frac{E\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)}{2}\left[\frac{A}{h}-\sum_{i\in\Gamma}c_i-\left(1-\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0-\sum_{i\in S}\frac{\lambda_i(z_i-1)T_i}{1-\lambda_i}\right]$$
$$\leq\ \frac{\lambda_j z_j T_j}{1-\lambda_j}-\left(\prod_{i\in\Gamma}\lambda_i\prod_{i\in S}z_i\right)\gamma_\Gamma^0,$$

which means $\partial B_S/\partial z_j \leq 0$, and thus $y_j^\Theta = 1$. ■

System profit—centralized vs. decentralized case. The maximum system profit under this scenario is $h\Psi(\mathbf{y}^\Theta)/4$. Hence, the system loses $(2n-2|S|+2\tau+1)^2/(2n-2|S|+2\tau+2)^2$ of its potential efficiency due to decentralized pricing decisions. Thus, as more farmers adopt tracing system (i.e., $|S|$ increases), the system profitability increases. However, as more coalitions are formed (i.e., τ increases), the system profitability reduces. Nonetheless, even if all farmers adopt the tracing system and form a single coalition (i.e., $|S| = n$ and $\tau = 1$), the system still loses 9/16 of its potential efficiency.

Coalition profit—centralized vs. decentralized case. We observe that the coalition S_l obtains $1/(n-|S|+\tau+1)^2$ of the centralized system profit. In the benchmark case where no product tracing occurs, the collective profit of the members in S_l amounts to $|S_l|/(n+1)^2$ of the centralized system profit. We find $1/(n-|S|+\tau+1)^2 \geq |S_l|/(n+1)^2$ if and only if $(n+1)^2 \geq |S_l|(n-|S|+\tau+1)^2$, i.e., $|S|-\tau$ is larger than a threshold. This indicates that coalition S_l can gain from adopting a product tracing system only if the majority of farmers participate in the tracing system and the number of coalitions formed remains relatively limited.

Coalition profit in decentralized model—with and without product tracing. Since $\Psi(\mathbf{y}^{\Theta}) \geq \Psi(\mathbf{0})$ and $n+1 \geq n-|S|+\tau+1$, we find $\pi_i^{\Theta} \geq \pi_i^{\emptyset}$, $\forall i \in \Gamma\backslash S$. This implies that product tracing in the farmer tier can improve the profits of non-tracing members, but the system as a whole still experiences substantial efficiency losses compared to the centralized setting. Thus, product tracing at the farmer's site provides only limited benefits for both the overall system and individual members.

Moreover, the total profit share of all coalitions collectively, i.e., $S = \cup_{k=1}^{\tau} S_k$, is $4\tau/(4n-4|S|+4\tau+3)$ in the decentralized system, compared to their total share of $4|S|/(4n+3)$ in the benchmark case with no product tracing. Since $4\tau/(4n-4|S|+4\tau+3) \leq 4|S|/(4n+3)$, it follows that product tracing at the farmer tier reduces the share of total profit obtained by the tracing system-adopting members in the decentralized system-wide profit.

Tracing decisions. All else being equal, if a farmer's unit identification cost is sufficiently high, the farmer will trace no products, whereas if the cost is sufficiently low, the farmer will trace all products. A farmer's unit identification cost comprises two components: production cost and tracing cost. A high production cost can discourage a farmer from tracing products because, instead of removing a deteriorated product with a high cost, the farmer might prefer to sell it downstream. Conversely, lower production costs make tracing more feasible.

In addition to unit identification costs, other factors influencing farmers' tracing decisions include quality elasticity of demand, system profitability, and penalty costs. For instance, if the system-wide penalty cost, γ_{Γ}^{0}, is sufficiently high, farmers in a coalition are likely to trace their products. However, the absence of penalty costs does not necessarily prevent tracing. Farmers may still trace products if the system profitability, $A/h - \phi(0)$, exceeds their unit identification costs (T_{Θ}) and the quality elasticity of demand is sufficiently large. Conversely, when both system profitability and penalty costs are low, farmers in coalitions are unlikely to trace any products.

4.3 The Coalition Structure When Only the Retailer Does Not Trace the Products ($S \cap \{M, R\} = \{M\}$)

We find equilibrium decisions under coalition structure Θ when the manufacturer adopts the tracing system but the retailer does not, i.e., $M \in S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma\backslash\{R\}$. Without loss of generality, we let $M \in S_1$ and define

$$T_M \equiv \left(\frac{1 - \prod_{i \in \Gamma\backslash\{R\}} \lambda_i}{\prod_{i \in \Gamma\backslash\{R\}} \lambda_i} \right) \sum\nolimits_{i \in \Gamma\backslash\{R\}} c_i + \frac{t_M}{\prod_{i \in \Gamma\backslash\{R\}} \lambda_i}.$$

We can view T_M as the manufacturer's net unit cost for identifying and removing a deteriorated product at its site, which is referred to as the manufacturer's *unit identification cost*. In addition, under coalition structure Θ, we compute unit identification cost T_Θ as

$$T_\Theta \equiv T_M + \frac{\sum_{i\in S\backslash\{M\}} T_i}{\prod_{i\in\Gamma\backslash\{R\}} \lambda_i}. \tag{10}$$

Proposition 3 *In a coalition structure $\Theta = \{S_1, \ldots, S_\tau\}$ such that $S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma\backslash\{R\}$ and $M \in S_1$, we find optimal tracing decisions as $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\}, i\in S} \Psi(\mathbf{y})$ ($y_i = 0$ for $i \in \Gamma\backslash S$).*

1. *For the manufacturer, if*

$$\frac{E_{\min}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta\right]^+ \geq T_\Theta + \frac{T_M - \left(\prod_{i\in S\backslash\{M\}} \frac{1}{\lambda_i} - 1\right)\sum_{i\in\Gamma\backslash\{R\}} c_i}{\prod_{i\in\Gamma\backslash\{R\}} \frac{1}{\lambda_i} - \prod_{i\in S\backslash\{M\}} \frac{1}{\lambda_i}} - \gamma_\Gamma^0 \prod_{i\in\Gamma} \lambda_i,$$

the manufacturer traces products (i.e., $y_M^\Theta = 1$); if

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0})\right] \leq \frac{T_M \prod_{i\in\Gamma\backslash\{R\}} \lambda_i}{1 - \prod_{i\in\Gamma\backslash\{R\}} \lambda_i} - \left(1 + \frac{E_{\max}}{2}\left(1 - \prod_{i\in\Gamma\backslash\{R\}} \lambda_i\right)\right)\lambda_R \gamma_\Gamma^0,$$

the manufacturer does not trace products (i.e., $y_M^\Theta = 0$).

2. *For farmer $j \in S\backslash\{M\}$, if*

$$\frac{\lambda_j T_j}{1-\lambda_j} \leq \min\left\{\frac{\lambda_j E_{\min}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta - \sum_{i\in\Gamma\backslash\{R\}} c_i\right]^+ + \gamma_\Gamma^0 \prod_{i\in\Gamma}\lambda_i, \prod_{i\in\Gamma}\lambda_i \sum_{i\in\Gamma\backslash\{R\}} c_i\right\},$$

the farmer traces products (i.e., $y_j^\Theta = 1$); if

$$\begin{aligned}\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0})\right] \quad \leq \quad & \frac{\lambda_j T_j}{1-\lambda_j} - T_\Theta \prod_{i\in\Gamma\backslash\{R\}} \lambda_i - \prod_{i\in\Gamma\backslash\{R\}} \lambda_i \sum_{i\in\Gamma\backslash\{R\}} c_i \\ & -\gamma_\Gamma^0 \lambda_R \left(1 + \frac{E_{\max}}{2}\left(1 - \prod_{i\in\Gamma\backslash\{R\}} \lambda_i\right)\right),\end{aligned}$$

farmer j does not trace products (i.e., $y_j^\Theta = 0$).

The corresponding profits of coalitions and other firms are

$$\begin{aligned}\pi_{S_1}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{8(n-|S|+\tau+1)^2};\ \pi_{S_l}^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } l \in \{2,\ldots,\tau\};\\ \pi_i^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } i \in (\Gamma\backslash\{R\})\backslash S;\ \pi_R^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{16(n-|S|+\tau+1)^2}.\end{aligned}$$

Proof. Since the retailer does not adopt the tracing system, we do not need to determine γ_M and $y_R = 0$. For this case $\varphi(\mathbf{y}) \equiv 1 - y_M + y_M \lambda_M \prod_{k=1}^{n} \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$ and the farmers in coalition S_1 do not need to determine their prices. The calculations of the equilibrium decisions from stage six to stage four are very similar to the proof of Proposition 1. Thus we omit the detailed calculations and list the profits of all coalitions and members that do not adopt the tracing system under given $\mathbf{y}$ as

$$\begin{aligned}\pi_{S_1} &= \frac{h\Psi(\mathbf{y})}{8(n-|S|+\tau+1)^2};\ \pi_{S_l} = \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } l \in \{2, ..., \tau\};\\ \pi_R &= \frac{h\Psi(\mathbf{y})}{16(n-|S|+\tau+1)^2};\ \pi_i = \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } i \in (\Gamma\backslash\{R\})\backslash S.\end{aligned}$$

These profit are all independent of the $\boldsymbol{\gamma}$, which means that decisions $\boldsymbol{\gamma}$ and $\mathbf{p}$ are correlated and $\boldsymbol{\gamma}$ is redundant. As a consequence, no calculation is needed in stage three.

In the second stage, all coalitions make their tracing decisions simultaneously. Since they all maximize the same objective $\Psi(\mathbf{y})$, the products tracing decisions satisfy $\mathbf{y}^{\Theta} = \arg\max_{y_i \in \{0,1\}, i \in S} \Psi(\mathbf{y})$ and we can write the profits of the coalition and all firms as in this proposition. Similar to the proof of Proposition 2, when the conditions in Proposition 3 hold, $\Psi(\mathbf{y})$ is always increasing (or decreasing) for all $y_j \in [0,1]$ ($j \in S$), which leads to $y_j^{\Theta} = 1$ (or $y_j^{\Theta} = 0$). ■

System profit—centralized vs. decentralized case. When firms from the two tiers form a coalition to trace products, the system incurs an efficiency loss of $(2n-2|S|+2\tau+1)^2/(2n-2|S|+2\tau+2)^2$ as a result of decentralized pricing decisions, which is the same as Proposition 2.

Coalition profit—centralized vs. decentralized case. The two-tier coalition S_1 obtains $1/[2(n-|S|+\tau+1)^2]$ of the centralized system profit. In contrast, under the benchmark case with no product tracing, the total profits of the members in S_1 amounts to $(2|S_1|-1)/[2(n+1)^2]$ of the centralized system profit. The condition $1/[2(n-|S|+\tau+1)^2] \geq (2|S_1|-1)/[2(n+1)^2]$ holds if and only if $(n+1)^2 \geq (2|S_1|-1)(n-|S|+\tau+1)^2$, which requires $|S|-\tau$ to exceed a threshold. This indicates that coalition S_1 benefits from product tracing only when the majority of farmers adopt the tracing system and the number of coalitions formed is relatively small.

Coalition profit in decentralized model—with and without product tracing. Similar to Proposition 2, the total profit obtained jointly by all members in S is $(4\tau-2)/(4n-4|S|+4\tau+3)$ of the decentralized system profit, which is lower than their share $(4|S|-2)/(4n+3)$ in the benchmark case.

Tracing decisions. The manufacturer's decision to trace products is influenced by several factors: the quality elasticity of demand, the unit identification costs of coalition members, the system's penalty cost, and overall profitability. If the penalty cost or system profitability is sufficiently high, the manufacturer opts to trace all products. Similarly, when the system profitability exceeds the unit identification cost T_Θ under coalition structure Θ, and the quality elasticity of demand is high, the manufacturer traces products. However, if the manufacturer's unit identification cost is substantial, no products are traced.

For farmers, high penalty cost (γ_Γ^0) and production cost ($\Sigma_{i\in\Gamma\backslash\{R\}}c_i$) encourage tracing, as high penalty incentivize coalitions to remove deteriorated products, and high production cost making it economically advantageous to remove at the farmer's tier. However, due to

$$\frac{\lambda_j T_j}{1-\lambda_j} - T_\Theta \prod_{i\in\Gamma\backslash\{R\}} \lambda_i = \frac{\lambda_j T_j}{1-\lambda_j} - T_M \prod_{i\in\Gamma\backslash\{R\}} \lambda_i - \sum_{i\in S\backslash\{M\}} T_i,$$

if a farmer's unit identification cost is high, the farmer refrains from tracing.

4.4 The Coalition Structure When Only the Manufacturer Does Not Trace the Products ($S \cap \{M, R\} = \{R\}$)

We now examine the scenario where, in the coalition structure Θ, the retailer (R) is part of $S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma\backslash\{M\}$. Without loss of generality, we assume $R \in S_1$ and define

$$T_R \equiv \frac{1-\prod_{i\in\Gamma}\lambda_i}{\prod_{i\in\Gamma}\lambda_i}\sum\nolimits_{i\in\Gamma} c_i + \frac{t_R}{\prod_{i\in\Gamma}\lambda_i},$$

which is the retailer's *unit identification cost.* For such coalition structure,

$$T_\Theta = T_R + \left(\sum\nolimits_{i\in S\backslash\{R\}} T_i\right) / \left(\prod\nolimits_{i\in\Gamma}\lambda_i\right).$$

Proposition 4 *In a coalition structure* $\Theta = \{S_1, \ldots, S_\tau\}$ *such that* $S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma\backslash\{M\}$ *and* $R \in S_1$*, we obtain optimal tracing decisions* $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\}, i\in S}\Psi(\mathbf{y})$ *(*$y_i = 0$ *for* $i \in \Gamma\backslash S$*).*

1. *For the retailer, if*

$$\frac{E_{\min}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta\right]^+ \geq T_\Theta + \frac{T_R - \left(\frac{1}{\prod_{i\in S\backslash\{R\}}\lambda_i} - 1\right)\sum_{i\in\Gamma} c_i}{\frac{1}{\prod_{i\in\Gamma}\lambda_i} - \frac{1}{\prod_{i\in S\backslash\{R\}}\lambda_i}} - \gamma_\Gamma^0 \prod_{i\in\Gamma}\lambda_i,$$

the retailer traces products (i.e., $y_R^\Theta = 1$*); if*

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0})\right] \leq \frac{T_R\prod_{i\in\Gamma}\lambda_i}{1-\prod_{i\in\Gamma}\lambda_i} - \left(1 + \frac{E_{\max}}{2}\left(1 - \prod_{i\in\Gamma}\lambda_i\right)\right)\gamma_\Gamma^0,$$

the retailer does not trace products (i.e., $y_R^\Theta = 0$).

2. *For farmer $j \in S\backslash\{R\}$, if*

$$\frac{\lambda_j T_j}{1-\lambda_j} \leq \min\left\{\prod_{i\in\Gamma}\lambda_i \sum_{i\in\Gamma} c_i, \gamma_\Gamma^0 \prod_{i\in\Gamma}\lambda_i + \frac{\lambda_j E_{\min}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - T_\Theta\right]^+\right\},$$

farmer j traces products (i.e., $y_j^\Theta = 1$); if

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0})\right] \leq \frac{\lambda_j T_j}{1-\lambda_j} - T_\Theta \prod_{i\in\Gamma}\lambda_i - \prod_{i\in\Gamma}\lambda_i \sum_{i\in\Gamma} c_i - \left(1 + \frac{E_{\max}}{2}\left(1 - \prod_{i\in\Gamma}\lambda_i\right)\right)\gamma_\Gamma^0,$$

farmer j does not trace products (i.e., $y_j^\Theta = 0$).

The corresponding profits of coalitions and other firms are

$$\begin{aligned}
\pi_{S_1}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{16(n-|S|+\tau+1)^2};\ \pi_{S_l}^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } l \in \{2,...,\tau\};\\
\pi_M^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{8(n-|S|+\tau+1)^2};\ \pi_i^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } i \in (\Gamma\backslash\{M\})\backslash S.
\end{aligned}$$

Proof. Since the manufacturer does not adopt the tracing system, we do not need to determine $\{\gamma_i\}_{i\in\Gamma\backslash\{M,R\}}$ and $y_M = 0$. For this case $\varphi(\mathbf{y}) \equiv 1 - y_R + y_R\lambda_R\lambda_M\prod_{k=1}^n \frac{\lambda_{F_k}}{1-y_{F_k}(1-\lambda_{F_k})}$. The calculations of the equilibrium decisions from stage six to stage four are very similar to the proof of Proposition 1. Thus we omit the detailed calculations and list the profits of all coalitions and members that do not adopt the tracing system under given $\mathbf{y}$ as

$$\begin{aligned}
\pi_{S_1} &= \frac{h\Psi(\mathbf{y})}{16(n-|S|+\tau+1)^2};\ \pi_{S_l} = \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } l \in \{2,...,\tau\};\\
\pi_M &= \frac{h\Psi(\mathbf{y})}{8(n-|S|+\tau+1)^2};\ \pi_i = \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } i \in (\Gamma\backslash\{M\})\backslash S.
\end{aligned}$$

Therefore, in the second stage, the products tracing decisions satisfy $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\},i\in S}\Psi(\mathbf{y})$. Similar to the proof of Proposition 2, when the conditions in Proposition 4 hold, $\Psi(\mathbf{y})$ is always increasing (or decreasing) for all $y_j \in [0,1]$ ($j \in S$), i.e., $y_j^\Theta = 1$ (or $y_j^\Theta = 0$). ■

Coalition profit—centralized vs. decentralized case. Proposition 4 demonstrates that the farmers and the retailer in coalition S_1 can secure $1/[4(n-|S|+\tau+1)^2]$ of the centralized system profit. Comparatively, in the benchmark case with no product tracing, their share of the centralized system profit is $(4|S_1|-3)/[4(n+1)^2]$. The condition $1/[4(n-|S|+\tau+1)^2] \geq (4|S_1|-3)/[4(n+1)^2]$ holds if and only if $(n+1)^2 \geq (4|S_1|-3)(n-|S|+\tau+1)^2$, which implies that $|S|-\tau$ must exceed a certain threshold. Thus, the farmer-retailer-tier coalition S_1 can benefit from cooperation if the majority of the farmers adopt the tracing system and the number of formed coalitions remains relatively small.

Coalition profit in decentralized model—with and without product tracing. Similar to Proposition 2, the total profit obtained jointly by all members in S is $(4\tau - 3)/(4n - 4|S| + 4\tau + 3)$ of the decentralized system profit, which is lower than their share $(4|S| - 3)/(4n + 3)$ in the benchmark case.

Tracing decisions. Consumers are guaranteed to receive only high-quality products if all products are traced at the retailer's site (i.e., $y_R^\Theta = 1$). Proposition 4 indicates that the retailer is incentivized to trace products when system profitability or the whole system's penalty cost is sufficiently high.

4.5 The Coalition Structure When the Manufacturer and the Retailer Adopt the Tracing System ($\{M, R\} \subset S$)

We first analyze the equilibrium decisions under coalition structure Θ where both the retailer and the manufacturer adopt the tracing system but they belong to separate coalitions. Specifically, $R, M \in S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma$ and $\{R, M\} \nsubseteq S_k$ for all $k \in \{1, \ldots, \tau\}$. Without loss of generality, we assume $R \in S_1$ and $M \in S_2$.

Proposition 5 *When $S = \cup_{k=1}^{\tau} S_k \subseteq \Gamma$, $R \in S_1$, and $M \in S_2$, we obtain optimal tracing decisions $\mathbf{y}^\Theta = \arg\max_{y_i \in \{0,1\}, i \in S} \Psi(\mathbf{y})$ ($y_i = 0$ for $i \in \Gamma \backslash S$). Moreover,*

1. *if*
$$\frac{t_j}{1-\lambda_j} \le \min\left\{t_M + \sum\nolimits_{i \in \Gamma \backslash \{R,j\}} c_i, t_R + \sum\nolimits_{i \in \Gamma \backslash \{j\}} c_i, \gamma_\Gamma^0 \lambda_j \lambda_M \lambda_R - c_j\right\},$$
then farmer $j \in S \backslash \{M, R\}$ traces products (i.e., $y_j^\Theta = 1$);

2. *if*
$$\frac{t_M}{1 - \prod_{i \in \Gamma \backslash [S \backslash \{M,R\}]} \lambda_i} \le \min\left\{c_R \prod_{i \in S \backslash \{M,R\}} \lambda_i, \gamma_\Gamma^0 \prod_{i \in \Gamma} \lambda_i - \sum_{i \in S \backslash \{M,R\}} T_i - \sum_{i \in \Gamma \backslash \{R\}} c_i\right\},$$
the manufacturer traces products (i.e., $y_M^\Theta = 1$);

3. *if*
$$\gamma_\Gamma^0 \prod_{i \in \Gamma} \lambda_i \;\ge\; \max\left\{\left(\frac{t_R}{1-\lambda_R} + c_R\right) \prod_{i \in \Gamma \backslash [S \backslash \{M\}]} \lambda_i + t_M, \frac{t_R}{1-\lambda_R} + c_R\right\} + \sum_{i \in \Gamma \backslash [S \backslash \{M\}]} c_i + \sum_{i \in S \backslash \{M,R\}} \frac{c_i + t_i}{\lambda_i}. \tag{11}$$

the retailer traces products (i.e., $y_R^\Theta = 1$).

The corresponding profits of coalitions and other firms are

$$\begin{aligned}
\pi_{S_1}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{16(n-|S|+\tau+1)^2};\ \pi_{S_2}^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{8(n-|S|+\tau+1)^2};\\
\pi_{S_l}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } l\in\{3,...,\tau\};\ \pi_i^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+1)^2},\ \textit{for } i\in\Gamma\backslash S.
\end{aligned}$$

Proof. The calculations of the equilibrium decisions from stage six to stage four are very similar to the proof of Proposition 1. Thus we omit the detailed calculations and list the profits of all coalitions and members that do not adopt the tracing system under given $\mathbf{y}$ as

$$\begin{aligned}
\pi_{S_1} &= \frac{h\Psi(\mathbf{y})}{16(n-|S|+\tau+1)^2};\ \pi_{S_2} = \frac{h\Psi(\mathbf{y})}{8(n-|S|+\tau+1)^2};\\
\pi_{S_l} &= \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } l\in\{3,...,\tau\};\ \pi_i = \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+1)^2},\ \text{for } i\in\Gamma\backslash S.
\end{aligned}$$

Therefore, in the second stage, the products tracing decisions satisfy $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\}}\Psi(\mathbf{y})$ ($i\in S$; $y_i=0$ for $i\in\Gamma\backslash S$). Note that

$$\Psi(\mathbf{y}) = \left[\max\left(0, \frac{A}{h} - \frac{\phi(\mathbf{y})}{\varphi(\mathbf{y})}\right)\right]^2 D\left(1 - \frac{\prod_{i\in\Gamma}\lambda_i}{\varphi(\mathbf{y})\prod_{i\in S\backslash\{M,R\}}[1-y_i(1-\lambda_i)]}\right),$$

and $\varphi(\mathbf{y})\prod_{i\in S\backslash\{M,R\}}[1-y_i(1-\lambda_i)]$ is increasing in y_i for all $i\in S$. As a consequence, $y_j^\Theta = 1$ ($j\in S$) if $\partial\left[\phi(\mathbf{y})/\varphi(\mathbf{y})\right]/\partial y_j \le 0$ for all $i\in S$, $y_i\in\{0,1\}$. Similar to the proof of Proposition 2, when the conditions in Proposition 5 hold, $\phi(\mathbf{y})/\varphi(\mathbf{y})$ is increasing (or decreasing) for all $y_j\in[0,1]$ ($j\in S$), i.e., $y_j^\Theta = 1$. ■

Next, we analyze the equilibrium decisions when both the retailer and the manufacturer adopt the tracing system and belong to the same coalition. Specifically, $R, M\in S = \cup_{k=1}^{\tau}S_k \subseteq \Gamma$ and $\{R,M\}\subseteq S_k$ for $k\in\{1,\dots,\tau\}$. Without loss of generality, we assume that $\{R,M\}\in S_1$.

Proposition 6 *When $S = \cup_{k=1}^{\tau}S_k\subseteq\Gamma$ and $\{R,M\}\subseteq S_1$, we find optimal tracing decisions $\mathbf{y}^\Theta = \arg\max_{y_i\in\{0,1\},i\in S}\Psi(\mathbf{y})$ ($y_i = 0$ for $i\in\Gamma\backslash S$). The results of tracing decisions are the same as in Proposition 5. The corresponding profits of coalitions and other firms are*

$$\begin{aligned}
\pi_{S_1}^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{4(n-|S|+\tau+2)^2};\ \pi_{S_l}^\Theta = \frac{h\Psi(\mathbf{y}^\Theta)}{2(n-|S|+\tau+2)^2},\ \textit{for } l\in\{2,...,\tau\};\\
\pi_i^\Theta &= \frac{h\Psi(\mathbf{y}^\Theta)}{2(n-|S|+\tau+2)^2},\ \textit{for } i\in\Gamma\backslash S.
\end{aligned}$$

Proof. The calculations of the equilibrium decisions from stage six to stage four are very similar to the proof of Proposition 1. Thus we omit the detailed calculations and list the profits of all coalitions and members that do not adopt the tracing system under given $\mathbf{y}$ as

$$\begin{aligned}\pi_{S_1} &= \frac{h\Psi(\mathbf{y})}{4(n-|S|+\tau+2)^2};\ \pi_{S_l} = \frac{h\Psi(\mathbf{y})}{2(n-|S|+\tau+2)^2}, \text{ for } l \in \{2,...,\tau\};\\ \pi_i &= \frac{h\Psi(\mathbf{y})}{2(n-|S|+\tau+2)^2}, \text{ for } i \in \Gamma\backslash S.\end{aligned}$$

Therefore, in the second stage, the products tracing decisions satisfy $\mathbf{y}^{\Theta} = \arg\max_{y_i\in\{0,1\},i\in S} \Psi(\mathbf{y})$. The proofs of the results for $\mathbf{y}^{\Theta}$ are similar to those for Proposition 5. ■

Similar to Proposition 4, the retailer traces all products if the penalty cost, γ_{Γ}^{0}, is sufficiently high. Since $\lambda_i \in (0,1)$ and $(c_i+t_i)/\lambda_i \geq c_i$, the right hand side of the condition in (11) increases as set S gets larger. This implies that, as more firms adopt tracing, the retailer is less likely to trace products because the proportion of deteriorated products at the retailer's site decreases, reducing the retailer's incurred penalties. Additionally, the food processing system's incentive to trace products increases when the production and/or tracing costs of all firms are reduced.

However, for tracing decisions at the farmer's or the manufacturer's sites, the conditions extend beyond a high system-wide penalty cost. For instance, at the manufacturer's site, an additional condition can be expressed as $t_M \leq c_R(\Pi_{i\in S\backslash\{M,R\}}\lambda_i - \Pi_{i\in\Gamma}\lambda_i)$. This implies that the manufacturer's tracing cost, t_M, must be sufficiently lower than a proportion of the retailer's production cost, c_R, to make tracing viable at the manufacturer's site.

Similarly, at the farmer's site, the condition for tracing can be expressed as

$$\frac{t_j}{1-\lambda_j} \leq \min\left\{t_M + \sum\nolimits_{i\in\Gamma\backslash\{R,j\}} c_i, t_R + \prod\nolimits_{i\in\Gamma\backslash\{j\}} c_i\right\}.$$

This implies that the tracing cost for farmer j must be lower than: (i) the sum of the manufacturer's processing cost after deducting farmer j's production cost ($\Sigma_{i\in\Gamma\backslash\{R,j\}}c_i$) plus tracing cost t_M, and (ii) the sum of the retailer's processing cost after farmer j's production cost ($\Sigma_{i\in\Gamma\backslash\{j\}}c_i$) is deducted plus tracing cost t_R. Thus, the whole system's penalty cost plays a role in each tier, and we need to account for the processing costs of downstream members in the food processing system.

We explain our above findings as follows. A sufficiently high penalty cost γ_{Γ}^{0} incentivizes the system to detect and remove all the deteriorated products. High downstream processing costs further encourage upstream firms to remove deteriorated products before passing them downstream, thereby reducing unnecessary expenses. In high "value-added" food processing systems, where all tiers have high processing costs, an increase in the retailer's penalty cost can encourage system-wide adoption of tracing, not just at the retailer's site.

The system experiences different levels of efficiency loss depending on the coalition structure. When the manufacturer and the retailer are not in the same coalition, the system efficiency loss is $(2n-2|S|+2\tau+1)^2/(2n-2|S|+2\tau+2)^2$. However, when the manufacturer and the retailer join the same coalition, the system efficiency loss is $(n-|S|+\tau+1)^2/(n-|S|+\tau+2)^2$. When the manufacturer and retailer trace products within the same coalition, the system becomes significantly more efficient. Figure 1 highlights the system efficiency under different scenarios: as the number of coalitions (τ) approaches its maximum ($|S|$), efficiency loss is 90% without cooperation and 80% with cooperation. For smaller τ efficiency losses reduce to 70% when neither firm adopts tracing and to 55% when both firms trace products. These results underscore the importance of collaboration between the manufacturer and retailer in improving system efficiency through product tracing.

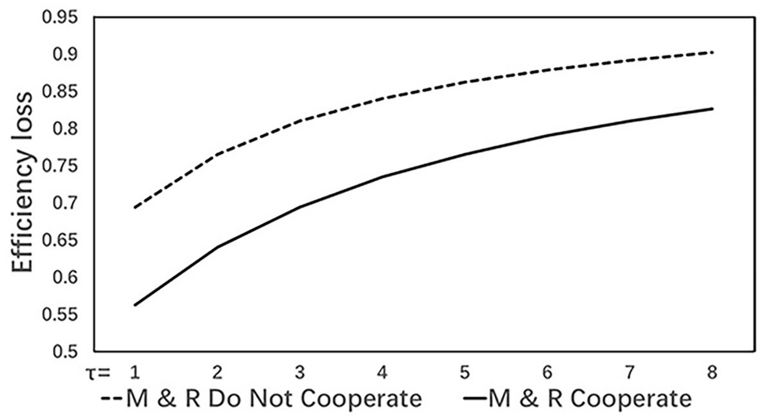

Figure 1: The efficiency loss in the food processing system ($n = 10$ and $|S| = 9$).

5 Cooperative Game Analysis

We develop a multi-player coalitional game $G = (\Gamma, v)$ in characteristic function form to model the food processing system, with $v : 2^\Gamma \to \mathbb{R}$. We proceed by forming the TU coalitional game $G = (\Gamma, v)$ based on our previous results.

5.1 The Product Tracing Game in the Food Processing System

For a non-empty coalition $S_k \subseteq \Gamma$, it is natural to define its characteristic value as the combined profit surplus of all the players within the coalition S_k. However, the total profit of the members in coalition S_k depends on the coalition structure $\Theta = \{S_1, \ldots, S_\tau\}$ such that $S_k \in \Theta$. As defined by, e.g., Straffin (1993), for our cooperative game the characteristic value of coalition S_k is the minimum surplus that the coalition can achieve, which means the surplus that all members in S_k can achieve under the worst coalition structure. Thus, the characteristic value of coalition S_k is expressed as

$$v(S_k) = \min_{\Theta \ni S_k} \pi_{S_k}^{\Theta} - \sum_{i \in S_k} \pi_i^{\emptyset} - |S_k|\Delta. \tag{12}$$

Proposition 7 *For a non-empty coalition $S_k \subseteq \Gamma$, $\arg\min_{\Theta \ni S_k} \pi_{S_k}^{\Theta} = \{S_k\}$.*

Proof. For a coalition $S_k \in \Theta$, since $\Psi(\mathbf{y}^{\Theta}) \geq \Psi(\mathbf{y}^{\{S_k\}})$ and $|S| - \tau \geq |S_k| - 1$, it is easy to verify $\pi_{S_k}^{\Theta} \geq \pi_{S_k}^{\{S_k\}}$ for the following four cases: (1) $S_k \cap \{M, R\} = \emptyset$; (2) $S_k \cap \{M, R\} = \{M\}$; (3) $S_k \cap \{M, R\} = \{R\}$; (4) $S_k \cap \{M, R\} = \{M\}$. ■

Proposition 7 indicates that a coalition's profit is minimized when all members outside the coalition do not adopt the tracing system. In addition, the specific formulation of $v(S_k)$ for different scenarios of coalition composition can be obtained as follows:

$$v(S_k) = \begin{cases} \dfrac{h\Psi(\mathbf{y}^{\{S_k\}})}{4(n - |S_k| + 2)^2} - \dfrac{|S_k| h\Psi(\mathbf{0})}{4(n+1)^2} - |S_k|\Delta, & \text{if } S_k \cap \{M, R\} = \emptyset; \\ \dfrac{h\Psi(\mathbf{y}^{\{S_k\}})}{8(n - |S_k| + 2)^2} - \dfrac{(2|S_k| - 1) h\Psi(\mathbf{0})}{8(n+1)^2} - |S_k|\Delta, & \text{if } S_k \cap \{M, R\} = \{M\}; \\ \dfrac{h\Psi(\mathbf{y}^{\{S_k\}})}{16(n - |S_k| + 2)^2} - \dfrac{(4|S_k| - 3) h\Psi(\mathbf{0})}{16(n+1)^2} - |S_k|\Delta, & \text{if } S_k \cap \{M, R\} = \{R\}; \\ \dfrac{h\Psi(\mathbf{y}^{\{S_k\}})}{4(n - |S_k| + 3)^2} - \dfrac{(4|S_k| - 5) h\Psi(\mathbf{0})}{16(n+1)^2} - |S_k|\Delta, & \text{if } S_k \cap \{M, R\} = \{M, R\}. \end{cases} \tag{13}$$

5.2 The Game Analysis

In this subsection, we discuss several game-theoretic concepts, with detailed definitions provided in online Appendix A. We begin by examining whether our coalitional game exhibits superadditivity and/or convexity.

Theorem 1 *For any two non-empty coalitions S_1 and S_2 such that $S_1 \cap \{M, R\} = \{M\}$ and $S_2 \cap \{M, R\} = \{R\}$, we find that $v(S_1 \cup S_2) + v(S_1 \cap S_2) \geq v(S_1) + v(S_2)$.*

Proof. For any two non-empty coalitions S_1 (with $s_1 = |S_1|$) and S_2 (with $s_2 = |S_2|$) such that $S_1 \cap \{M, R\} = \{M\}$ and $S_2 \cap \{M, R\} = \{R\}$, we let $S_\cap = S_1 \cap S_2$ (with $s_\cap = |S_\cap|$) and $S_\cup = S_1 \cup S_2$ (with $s_\cup = |S_\cup|$).

When $S_\cap = \emptyset$, $\max\{s_1, s_2\} \leq s_1 + s_2 - 1 = s_\cup - 1$ and $\max\{\Psi(\mathbf{y}^{\{S_1\}}), \Psi(\mathbf{y}^{\{S_2\}})\} \leq \Psi(\mathbf{y}^{\{S_\cup\}})$. Thus,

$$\begin{aligned}
& v(S_\cup) - v(S_1) - v(S_2) \\
= \; & \frac{h\Psi(\mathbf{y}^{\{S_\cup\}})}{4(n - s_\cup + 3)^2} - \frac{h\Psi(\mathbf{y}^{\{S_1\}})}{8(n - s_1 + 2)^2} - \frac{h\Psi(\mathbf{y}^{\{S_2\}})}{16(n - s_2 + 2)^2} \\
\geq \; & \frac{h\Psi(\mathbf{y}^{\{S_\cup\}})}{16(n - s_\cup + 3)^2} \geq 0.
\end{aligned}$$

When $S_\cap \neq \emptyset$, $\max\{s_1, s_2\} \leq s_1 + s_2 - 1 = s_\cup - 1$ and $\max\{\Psi(\mathbf{y}^{\{S_1\}}), \Psi(\mathbf{y}^{\{S_2\}})\} \leq \Psi(\mathbf{y}^{\{S_\cup\}})$. Therefore,

$$\begin{aligned}
& v(S_\cup) + v(S_\cap) - v(S_1) - v(S_2) \\
= \; & \frac{h\Psi(\mathbf{y}^{\{S_\cup\}})}{4(n - s_\cup + 3)^2} + \frac{h\Psi(\mathbf{y}^{\{S_\cap\}})}{4(n - s_\cap + 2)^2} - \frac{h\Psi(\mathbf{y}^{\{S_1\}})}{8(n - s_1 + 2)^2} - \frac{h\Psi(\mathbf{y}^{\{S_2\}})}{16(n - s_2 + 2)^2} \\
\geq \; & \frac{h\Psi(\mathbf{y}^{\{S_\cup\}})}{16(n - s_\cup + 3)^2} \geq 0.
\end{aligned}$$

■

As Theorem 1 illustrates, the condition for convexity is satisfied when the manufacturer and the retailer belong to different coalitions. Therefore, the collaborative product tracing approach benefits both the manufacturer and the retailer, regardless of whether the farmers decide to trace the products or not. While our coalitional game may not be inherently superadditive or convex, it can still possess a nonempty core.

Theorem 2 *The CIS-value*

$$CIS_i = v(i) + \frac{1}{|N|}\left(v(N) - \sum\nolimits_{j \in N} v(j)\right), \text{ for } i \in N, \tag{14}$$

is always in the core of our coalitional game. In addition, the CIS-value coincides with the nucleolus if $n \geq 3$.

Proof. For CIS-value CIS, we compute the excess ("unhappiness") for each coalition S_k under four cases: (1) $S_k \cap \{M, R\} = \emptyset$; (2) $S_k \cap \{M, R\} = \{M\}$; (3) $S_k \cap \{M, R\} = \{R\}$; (4) $S_k \cap \{M, R\} = \{M\}$.

For case (1) $S_k \cap \{M,R\} = \emptyset$, we have $1 \le |S_k| \le n$ and

$$\begin{aligned}
e(S_k, CIS) &= \frac{|S_k|}{n+2}\left[\sum_{i\in[\Gamma\backslash\{M,R\}]\backslash S_k}\frac{h\Psi(\mathbf{y}^{\{i\}})}{4(n+1)^2}+\frac{h\Psi(\mathbf{y}^{\{M\}})}{8(n+1)^2}+\frac{h\Psi(\mathbf{y}^{\{R\}})}{16(n+1)^2}-\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4}\right] \\
&\quad +\frac{h\Psi(\mathbf{y}^{\{S_k\}})}{4(n-|S_k|+2)^2}-\frac{n+2-|S_k|}{n+2}\sum_{i\in S_k}\frac{h\Psi(\mathbf{y}^{\{i\}})}{4(n+1)^2} \\
&\le \frac{|S_k|h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n+2)}\left[\frac{(4n-4|S_k|+3)}{4(n+1)^2}+\frac{n+2}{|S_k|(n-|S_k|+2)^2}-1\right] \le 0.
\end{aligned}$$

For other three cases, the proofs are very similar to case (1), and we omit the detailed calculations. Thus the CIS-value is in the core, and the core is non-empty.

The CIS-value coincides with the nucleolus if and only if the excesses for all single-player coalitions reach their maximum values under the CIS-value, i.e., $e(i,CIS) \ge e(S_k,CIS)$ for all $i \in \Gamma$ and $S_k \subsetneq \Gamma$ with $2 \le |S_k| \le n+1$. Since $e(i,CIS) = -[v(\Gamma)-\Sigma_{j\in\Gamma}v(j)]/n$ is constant for any $i \in \Gamma$, the CIS-value coincides with the nucleolus if and only if

$$\frac{1}{|S_k|-1}\left(v(S_k)-\sum_{j\in S_k}v(j)\right) \le \frac{1}{n+2}\left(v(\Gamma)-\sum_{j\in\Gamma}v(j)\right) \text{ for all } 2 \le |S_k| \le n+1.$$

In the following, we consider the following four cases of $S_k \cap \{M,R\}$. If $S_k \cap \{M,R\} = \emptyset$, then $2 \le |S_k| \le n$, and we have

$$\begin{aligned}
&\frac{1}{n+2}\left(v(\Gamma)-\sum_{j\in\Gamma}v(j)\right)-\frac{1}{|S_k|-1}\left(v(S_k)-\sum_{j\in S_k}v(j)\right) \\
=\ &\frac{1}{n+2}\left(\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4}-\sum_{j\in[\Gamma\backslash\{M,R\}]\backslash S_k}\frac{h\Psi(\mathbf{y}^{\{j\}})}{4(n+1)^2}-\frac{h\Psi(\mathbf{y}^{\{M\}})}{8(n+1)^2}-\frac{h\Psi(\mathbf{y}^{\{R\}})}{16(n+1)^2}\right) \\
&-\frac{1}{|S_k|-1}\frac{h\Psi(\mathbf{y}^{\{S_k\}})}{4(n-|S_k|+2)^2}+\left(\frac{1}{|S_k|-1}-\frac{1}{n+2}\right)\sum_{j\in S_k}\frac{h\Psi(\mathbf{y}^{\{j\}})}{4(n+1)^2} \\
\ge\ &\frac{1}{n+2}\left(\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4}-\sum_{j\in[\Gamma\backslash\{M,R\}]\backslash S_k}\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n+1)^2}-\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{8(n+1)^2}-\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{16(n+1)^2}\right) \\
&-\frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(|S_k|-1)(n-|S_k|+2)^2} \ge 0.
\end{aligned}$$

Similarly, we obtain the results for other three cases (i.e., $S_k \cap \{M,R\} = \{M\}$ or $\{R\}$ or $\{M,R\}$).

■

Theorem 2 implies that our coalitional game always possesses a non-empty core by identifying an allocation within the core that promotes the stability of the grand coalition. Under the CIS-value, a player with a higher individual contribution (IC) can obtain a larger share of the profit surplus generated by the grand coalition, $v(\Gamma)$, resulting from the centralization of product tracing and pricing decisions. Moreover, when $\Theta = \{j\}$ for some farmer $j \in \Gamma \backslash \{M, R\}$ in Proposition 2, we find that $y_j^{\{j\}} = 0$, $\pi_j^{\{j\}} = \pi_j^{\emptyset}$, and $v(j) = \min_{i \in \Gamma} v(i)$ if

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - \gamma_\Gamma^0 \prod_{i \in \Gamma} \lambda_i\right] + \gamma_\Gamma^0 \left(\frac{E_{\max}}{2} + 1\right) \prod_{i \in \Gamma \backslash \{j\}} \lambda_i \leq \frac{\lambda_j T_j}{1 - \lambda_j},$$

where, in the RHS, $\lambda_j T_j/(1-\lambda_j) = c_j + t_j/(1-\lambda_j)$. Because only $1-\lambda_j$ of products are deteriorated, farmer j incurs processing cost c_j and average tracing cost $t_j/(1-\lambda_j)$ when the farmer removes a deteriorated product. Therefore, we can view $\lambda_j T_j/(1-\lambda_j)$ as farmer j's *unit removing cost.* A farmer with a larger unit removing cost can receive a lower allocation under the CIS-value. Similarly, the manufacturer's and the retailer's unit removing costs are $T_M \Pi_{i \in \Gamma \backslash \{R\}} \lambda_i/(1 - \Pi_{i \in \Gamma \backslash \{R\}} \lambda_i)$ and $T_R \Pi_{i \in \Gamma} \lambda_i/(1 - \Pi_{i \in \Gamma} \lambda_i)$, respectively. When $\Theta = \{M\}$ in Proposition 3 and $\Theta = \{R\}$ in Proposition 4, we find that (1) $y_M^{\{M\}} = 0$, $\pi_M^{\{M\}} = \pi_M^{\emptyset}$, and $v(M) = \min_{i \in \Gamma} v(i)$, if

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - \gamma_\Gamma^0 \prod_{i \in \Gamma} \lambda_i\right] + \left(1 + \frac{E_{\max}}{2}\right) \lambda_R \gamma_\Gamma^0 \leq \frac{T_M \prod_{i \in \Gamma \backslash \{R\}} \lambda_i}{1 - \prod_{i \in \Gamma \backslash \{R\}} \lambda_i},$$

(2) $y_R^{\{R\}} = 0$, $\pi_R^{\{R\}} = \pi_R^{\emptyset}$, and $v(R) = \min_{i \in \Gamma} v(i)$, if

$$\frac{E_{\max}}{2}\left[\frac{A}{h} - \phi(\mathbf{0}) - \gamma_\Gamma^0 \prod_{i \in \Gamma} \lambda_i\right] + \left(1 + \frac{E_{\max}}{2}\right) \gamma_\Gamma^0 \leq \frac{T_R \prod_{i \in \Gamma} \lambda_i}{1 - \prod_{i \in \Gamma} \lambda_i}.$$

The above implies that a member with a large unit removing cost may receive the lowest allocation under the CIS-value. It is also interesting to note that the CIS-value coincides with the nucleolus when the number of farmers is larger than 3.

Although the CIS-value in (14) is a simple allocation for our coalitional game with some desirable properties, it does not guarantee that all players can receive a non-negative allocation. To incentivize the system-wide product tracing, each player should receive a positive allocation in the grand coalition (i.e., $\min_{i \in \Gamma} CIS_i > 0$) if the system-wide product tracing can benefit the grand coalition (i.e., $v(\Gamma) = \pi_\Gamma^{\{\Gamma\}} - \sum_{i \in \Gamma} \pi_i^{\emptyset} - (n+2)\Delta > 0$). To that end, we provide a new allocation, the *evenly-split* *value* or ES-value:

$$ES_i = \frac{v(\Gamma)}{n+2}, \text{ for } i \in \Gamma. \tag{15}$$

Theorem 3 *If the food processing system benefits from the system-wide product tracing (i.e., $v(\Gamma) > 0$), the ES-value is a core allocation that apportions a positive allocation to each firm.*

Proof. When $v(\Gamma) > 0$, it is trivial that $ES_i = v(\Gamma)/(n+2) > 0$. In the following, we show that $(ES_i)_{i\in\Gamma}$ is in the core of the game (Γ, v) under four cases of $S_k \cap \{M, R\}$. If $S_k \cap \{M, R\} = \emptyset$, we have

$$\begin{aligned}
& \sum_{i\in S_k} ES_i - v(S_k) \\
= \; & \frac{|S_k|h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n+2)} - \frac{|S_k|(4n+3)h\Psi(\mathbf{0})}{16(n+2)(n+1)^2} - \frac{h\Psi(\mathbf{y}^{\{S_k\}})}{4(n-|S_k|+2)^2} + \frac{|S_k|h\Psi(\mathbf{0})}{4(n+1)^2} \\
\geq \; & \frac{|S_k|h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n+2)} - \frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n-|S_k|+2)^2} - \frac{|S_k|(4n+3)h\Psi(\mathbf{0})}{16(n+2)(n+1)^2} + \frac{|S_k|h\Psi(\mathbf{0})}{4(n+1)^2} \\
\geq \; & \frac{h\Psi(\mathbf{0})}{4(n-|S_k|+2)^2}\left[\frac{(n-|S_k|+2)^2|S_k|}{4(n+2)}\left(4 + \frac{4(n+2)}{(n+1)^2} - \frac{4n+3}{(n+1)^2}\right) - 1\right] \geq 0.
\end{aligned}$$

Similarly, we can obtain the results for other three cases. ■

We observe that $v(\Gamma) > 0$ is equivalent to $\pi_\Gamma^\Gamma - \sum_{i\in\Gamma}\pi_i^\emptyset > (n+2)\Delta$, and

$$\frac{\pi_\Gamma^\Gamma - \sum_{i\in\Gamma}\pi_i^\emptyset}{n+2} = \frac{h\Psi(\mathbf{y}^{\{\Gamma\}})}{4(n+2)} - \frac{(4n+3)h\Psi(\mathbf{0})}{16(n+2)(n+1)^2} \geq \frac{(2n+1)^2\Psi(\mathbf{0})}{16(n+2)(n+1)^2} = \frac{(2n+1)^2}{(n+2)(4n+3)}\sum_{i\in\Gamma}\pi_i^\emptyset > 0.$$

Hence, all firms have incentives to jointly adopt product tracing if and only if the unit fixed cost (i.e., Δ) is below a specific threshold. Because $(2n+1)^2/[(n+2)(4n+3)] \geq 3/7$, this condition implies that firms will jointly trace products if Δ is less than 3/7 of the total profit achieved in the decentralized benchmark case without product tracing. Furthermore, as the number of farmers approaches infinity, the threshold for Δ approaches the total profit of all firms in the decentralized benchmark case, making joint product tracing increasingly viable as the scale of the system grows.

6 Conclusions

This paper presents a comprehensive analysis of strategic product tracing in a three-tier food processing system comprising multiple farmers, one manufacturer, and one retailer. We analyze the optimal decisions made by each member of the food processing system in a decentralized setting, considering different product-tracing scenarios. The members can form coalitions to adopt a tracing system and jointly make the tracing and pricing decisions.

We calculate the equilibrium results for all possible coalition structures. We find that the equilibrium tracing decision depends on several factors, including the quality elasticity of demand, unit identification costs, system-wide penalty costs, and overall profitability. For example, a firm in a coalition traces its products if the system-wide penalty cost is sufficiently large. However, a firm does not trace any product if its unit identification costs are significant. When the firms from different tiers form coalitions to adopt a tracing system, high processing costs incurred at one tier can encourage upstream partners to adopt tracing systems. Interestingly, the retailer's incentive to trace products diminishes as more firms adopt the tracing system, reflecting the interdependence of tracing decisions in the food processing system.

We also examine the implications of coalition formation. While coalitions may reduce the bargaining power of their members relative to independent firms, they can lead to higher individual profits when most farmers adopt tracing and the number of coalitions is relatively small. This is primarily due to an increase in system-wide profits. Notably, cooperation between the manufacturer and the retailer emerges as a particularly effective strategy, significantly enhancing system-wide profitability. A single, unified coalition proves to be the most beneficial configuration for the entire system.

Employing cooperative game theory, the study defines the characteristic value of a coalition as the profit surplus achievable under the worst-case scenario, which arises when all non-coalition firms abstain from tracing. We show that the CIS-value is always a core allocation and coincides with nucleolus when there are three or more farmers. However, the CIS-value may not always allocate positive surplus to firms when the whole system's profit surplus is positive. To address this limitation, we propose the evenly-split value (ES-value), offering an allocation method that guarantees a positive surplus for each firm while remaining within the core.

In summary, this research provides insights into the dynamics of product tracing in food processing systems, considering different tiers and cooperative scenarios. It offers practical allocation methods to encourage system-wide tracing while addressing the specific economic conditions of each player.

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Online Supplements

"**System-Wide Incentives to Trace Food Processing: A Cooperative-Game Analysis**"

F. Zhong, M. Leng, G. Sošić, M. Parlar

Appendix A Game-Theoretic Concepts

Two desirable properties of cooperative games are superadditivity and convexity. A coalitional game (N, v) is *superadditive* if $v(S \cup T) \geq v(S) + v(T)$ for any two disjoint coalitions $S, T \subset N$; for details, see, for example, Straffin (1993). A coalitional game's characteristic function is supermodular if $v(S \cup T) + v(S \cap T) \geq v(S) + v(T)$, for all S, $T \subseteq N$; (Shapley 1971). The game is *convex* if its characteristic function is supermodular.

A mapping Φ that assigns to every cooperative game (N, v) a point $\Phi(v) \in \mathbb{R}^{|N|}$ is called an *allocation rule*, and the value assigned by an allocation rule to a specific game (N, v) is referred to as an *allocation*.

In cooperative game theory, the *core* (Gillies 1959) is a crucial concept that characterizes a set of fair allocation rules ensuring the stability of the grand coalition. Let κ_i denote the profit allocated to firm $i, i \in N$. An *imputation* for a game (N, v) (Straffin (1993)) is an n-tuple $\kappa \equiv (\kappa_i)i \in N$ that satisfies two properties: (i) individual rationality, meaning $\kappa_i \geq v(i)$ for all players $i \in N$, and (ii) collective rationality, such that $\sum_{i \in N} \kappa_i = v(N)$. The core of a game consists of all undominated imputations, where for every coalition $T \subseteq N$, $\sum_{i \in T} \kappa_i \geq v(T)$. Consequently, any allocation within the core ensures the stability of the grand coalition. In other words, if the core of our coalitional game is non-empty, we can allocate the benefits from higher transparency among system members in a way that encourages a system-wide product tracing.

It is important to note that when a game is convex, its core is guaranteed to be non-empty. However, the core may contain an infinite number of imputations, making it necessary to identify a unique allocation that is both computationally feasible and acceptable to all members of the food processing system.

The *nucleolus* (Schmeidler 1969) is a member of the core whenever the core is non-empty. The nucleolus is defined as the unique allocation ν that lexicographically minimizes the vector of excesses $e(S, \nu) = v(S) - \sum_{i \in S} \nu_i$, where $S \subset N$, when these excesses are arranged in descending order.

Calculating the nucleolus can be complex for arbitrary games.

The *center of gravity of imputation set value* (CIS-value; see Driessen and Funaki (1997)) is an allocation scheme that allocates to each player in the game (N, v) an allocation defined as

$$CIS_i = v(i) + \frac{1}{|N|}\left(v(N) - \sum\nolimits_{j \in N} v(j)\right), \text{ for } i \in N.$$

Here, the allocation to each player consists of two parts. The first part is the player's *individual contribution* (IC) measured by $v(i)$, and the second part is the *egalitarian non-individual contribution* (ENIC), which ensures the equal allocation of the non-individual contribution among all players. This is calculated as $[v(N) - \sum_{i \in N} v(i)]/|N|$. For different definitions of IC, there are several forms of ENIC-based allocation schemes (Driessen and Funaki 1997).