Popper's Measure of Corroboration and $\mathrm{P}(h|b)$

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This paper shows that Popper's measure of corroboration is inapplicable if, as Popper also argued, the logical probability of synthetic universal statements is zero relative to any evidence that we might possess. It goes on to show that Popper's definition of degree of testability, in terms of degree of logical content, suffers from a similar problem.

1 *The Corroboration Function and* $P(h|b)$

2 *Degrees of Testability and* $P(h|b)$

1. The Corroboration Function and $\mathrm{P}(h|b)$

Popper ([1983], p. 240) proposes the following measure of corroboration, with *h* representing a universal scientific hypothesis, *e* representing a report on a test of *h*, and *b* representing background information assumed in performing the test:

$$(1) \qquad \mathrm{C}(h,e,b) = \frac{\mathrm{P}(e|hb) - \mathrm{P}(e|b)}{\mathrm{P}(e|hb) - \mathrm{P}(eh|b) + \mathrm{P}(e|b)}$$

Degree of corroboration is therefore 'nothing but a measure of the degree to which a hypothesis *h* has been tested, and of the degree to which it has stood up to tests.' (Popper [1959], p. 415). It is important theoretically since it 'is a means of stating *preference with respect to truth*' (Popper [1972], p. 20) and pragmatically

since 'we should *prefer* as basis for action the best-tested theory' (Popper [1972], p. 22).[1]

The probabilities are to be understood logically—with $\mathrm{P}(p)$ defined as $\mathrm{P}(p|T)$ where T is any tautology—as Popper ([1983], pp. 284–5) makes clear. However, Popper ([1959], Appendix *vii) also argues that the logical probability of any universal hypothesis is zero relative to any finite set of observation statements.[2] Thus if b only contains finitely many observation statements, e.g. statements of initial conditions, then:

$$(2) \qquad \mathrm{P}(h|b) = 0$$

Now we need only note, from the axioms of probability, that:

$$(3) \qquad \mathrm{P}(eh|b) = \mathrm{P}(e|hb)\mathrm{P}(h|b)$$

And we may conclude that if b only contains finitely many observation statements then:

$$(4) \qquad \mathrm{C}(h, e, b) = \frac{\mathrm{P}(e|hb) - \mathrm{P}(e|b)}{\mathrm{P}(e|hb) + \mathrm{P}(e|b)}$$

[1] Popper's views on the significance of corroboration underwent change throughout his career. For more detail, see Rowbottom [2010], Sections 2.4–2.5.3.

[2] In his own words, this is 'probability *relative to some evidence*; that is to say, relative to a singular statement, or to a finite conjunction of singular statements' (ibid.) Put simply, the idea is that infinitely many theories will be compatible with those observation statements and that those theories must be assigned equal probabilities. See Rowbottom [2010], Section 2.3.

In short, the central term on the denominator of (1), which is present for the purposes of normalization, is defunct. But (4) has various features that render it unsuitable as a function for measuring corroboration; chiefly, it doesn't even provide a suitable *ordering* of how well theories have fared in response to testing. Compare two scenarios in which e is found to be true, the first in which $\mathrm{P}(e|hb) = 1$ and $\mathrm{P}(e|b) = 0.1$, and the second in which $\mathrm{P}(e|hb) = 0.1$ and $\mathrm{P}(e|b) = 0.01$. According to (4), h is equally corroborated, i.e. has a corroboration value of 9/11, in each scenario.[3] This is patently absurd, however, since in the former scenario e is entailed by h and b (and discovery of $\sim e$ would have falsified the conjunct), whereas in the latter scenario h makes no notable contribution to predicting e in the presence of b (and discovery of $\sim e$ would hardly have been a blow for h and b).[4]

But can b contain something other than a finite number of observation statements, and in a way such that (2) is sometimes false? First, I take it that we cannot possess infinitely many observation statements. Second, since (1) cannot concern a test of h if h is entailed by b, for the simple reason that h would make no predictions above and beyond b, we must conclude that $\mathrm{P}(h|b) < 1$ whenever (1) is applicable.[5] This leaves only the possibility, third, that b can contain non-observation statements, e.g. universal statements such as scientific theories, which render h one of a finite number of alternatives and/or bear on h more favourably than other competing hypotheses compatible with b. Is this plausible?

[3] In short, (4) is sensitive only to the ratio of $\mathrm{P}(e|hb)$ to $\mathrm{P}(e|b)$.
[4] As Popper ([1983], p. 240) stated: 'The support given by e to h becomes significant only when… $p(e,hb) - p(e,b) >> ½$'.
[5] In the words of Popper ([1959], p. 418), his measure: 'can be interpreted as a degree of corroboration only if e is a report on the severest tests we have been able to design'.

Let *b** be the subset of *b* that contains no observation statements. And let's start by considering the scientific theories in *b**. They cannot be theories inconsistent with *h*, because (2) then holds provided that *h* and *b* are individually consistent. (For Popper, as for Carnap and Keynes, the logical view of probability was supposed to be an extension of classical logic.) So one natural way of thinking, e.g. that a theory like special relativity might be suggested by background knowledge of Newtonian mechanics, is precluded; since Newtonian mechanics is incompatible with special relativity, strictly speaking, *b** must not contain Newtonian mechanics in any test of special relativity. Of course, *b** may instead contain the information that the predictions made using Newtonian mechanics were successful (i.e. right within some error range) in a wide range of circumstances, i.e. for velocities low enough such that the gamma factor is approximately equal to one. (And this assumption of the approximate empirical adequacy of Newtonian mechanics in a peculiar class of circumstances *does* go beyond the finite number of observations available, i.e. *b*.) Yet this is still compatible with infinitely many theories other than *h*.

In short, the worry is that if Popper's argument that the probability of *h* relative to any finite number of observation statements is zero is successful, then it also shows that the probability of *h* relative to any infinite number of observation statements predicted by some theory (such as those implied by Newtonian mechanics) is zero when those only cover a limited range. View *h* as a curve. Consider observation statements—feel free to imagine if liked, as it makes no difference in the present context, that these are infallible—to be points on the curve. For any finite number of points, there are infinitely many curves that pass through. Now consider an infinite number of observation statements but only in a peculiar variable range. Here we will have a segment (or segments) of the curve *h*, but infinitely many curves

contain this segment (or these segments). Thus if the argument for the logical probability of *h* being zero works for finitely many observations, it works for infinitely many observations (or assumed-to-be-correct predictions) when these are in a limited range.[6] And if they aren't in a limited range, recall, then they will either entail *h* or conflict with *h*. But as we have seen above, (1) is only applicable when *b* neither entails *h* nor is inconsistent with *h*! (And if *b** entails *h* or is inconsistent with *h* then *b* entails *h* or is inconsistent with *h* because *b** is a subset of *b*.[7])

Perhaps there are other relevant items that could be placed in *b**? One idea, for instance, might be to introduce assumptions related to theoretical virtues, such as 'The simplest theory compatible with *b* is the most likely to be true'. Yet even if we assume that we have an appropriate measure of relative virtuosity—in this example, of simplicity—this strategy appears to go against the spirit of an anti-inductivist stance in the philosophy of science, and to question the relevance of Popper's argument for (2) in the first place. If we are free to help ourselves to this kind of assumption, then we will find that finitely many observation statements can grant high probabilities to theories. 'Prefer the simplest theories available which are compatible with the evidence' may indeed be a methodological rule for Popper, but this is far from suggesting that simplicity is a guide to truth or falsity (and therefore that

[6] Naturally the fact that observations are only made within some error range means that matters are worse than suggested here; in short, we do not have access to a segment but only a range of possible segments within the error bars.

[7] Donald Gillies suggested, in correspondence, that if we allow *h* to be a model rather than a theory (on a syntactic view of theories), e.g. of the Moon's motion, and then *b** may be understood to contain theories used in its construction, e.g. Newtonian mechanics, in a non-problematic fashion. This is an interesting idea, but would only work in limited contexts where two incompatible theories (such as Newtonian mechanics and relativity) were not being compared. Furthermore, it would constitute a departure from the Popperian emphasis on theory, and not merely model, testing.

simplicity has anything to do, whatsoever, with corroboration, confirmation, or falsification).

2. Degree of Testability and $\mathrm{P}(\boldsymbol{h}|\boldsymbol{b})$

But so what, if (1) becomes (4) and (4) is unfit for purpose? Since the denominator of (1) is only supposed to fulfill a normalizing role, one might maintain that the workhorse of the equation, namely the numerator, is of intuitive significance. One might add that Popper ([1983], p. 242) notes: 'certain logarithmic formulae may do just as well – or better for certain purposes'. This is fair. But it is important to note that if (1) is abandoned for the reasons above then one should also abandon one of the key ideas behind its introduction, namely that *degree of testability* is equal to $1 - \mathrm{P}(h|b)$. Popper ([1983], p. 241) expresses this idea as follows:

> [I]f $p(h,b)\neq 0$, the maximum value which $C(h,e,b)$ can attain is equal to $1 - p(h,b)$ and therefore equal to the content of h relative to b, or to its degree of testability. This makes the degree of testability equal to the maximal degree of corroboration of h, or to its 'degree of corroborability'.

Prima facie, as a result of the argument above, one might conclude that all universal theories are equally testable. But this would not be a happy result for Popper, given what he says in chapter six of *The Logic of Scientific Discovery*, because it would mean that two universal hypotheses were equally testable irrespective of their relative empirical content. Allow me to explain.

Popper ([1959], Chapter vi) distinguishes between two forms of content, empirical and logical. In his own words:

> I define the *empirical content* of a statement *p* as the class of its potential falsifiers. The *logical content* is defined… as the class of all non-tautological statements which are derivable from the statement in question. (Popper [1959], p. 120)

Popper then relates degree of testability to degree of empirical content; and he argues that degrees of testability *qua* degrees of empirical content are crucial in theory-choice. As the well-known slogan goes, we should prefer bold hypotheses; bold, that is, precisely in so far as easily falsifiable (and therefore highly testable).[8] Consider, for example, "All swans are white" versus "All swans are white or black". The former has potential falsifiers which the latter does not, and is therefore intuitively more testable.

Popper continues by relating logical content to empirical content (and therefore testability), i.e. the consequence class of a statement (minus tautologies) with the class of its potential falsifiers, in a variety of fashions. He arrives at the following thesis:

> In comparing degrees of testability or of empirical content we shall… as a rule – i.e. in the case of purely empirical statements – arrive as the same results as in comparing logical content, or derivability relations. (Popper [1959], p. 121)

[8] Popper ([1959], p. 112) also equates degree of testability with degree of falsifiablilty: 'Theories may be more, or less, severely testable; that is to say, more, or less, easily falsifiable'.

We should consider Popper's comments about the measure $1 - \mathrm{P}(h|b)$ in this light. As we have seen above, he equates the measure both to 'degree of testability' and 'the content of *h* relative to *b*' (Popper [1983], p. 241). The content in question here is presumably logical, in so far the measure is defined in terms of logical probability; i.e. $1 - \mathrm{P}(h|b)$ is supposed to be a measure of the *logical* 'content of *h* relative to *b*' (ibid.) Thus it is plausible that Popper has the following thesis in mind:

($) Degree of empirical content of *h* relative to *b* is equal to degree of logical content of *h* relative to *b*.

Even if this is wrong, and Popper does not think ($) is true, he writes of 'degree of testability or of empirical content' (Popper [1959], p. 121) and states that $1 - \mathrm{P}(h|b)$ is also equal to 'degree of testability' (Popper [1983], p. 241).[9] Hence he thinks that:

(£) Degree of empirical content of *h* relative to *b* is equal to $1 - \mathrm{P}(h|b)$.

In his own words: '*corroborability equals testability and empirical content*' (Popper [1983], p. 245) [emphasis in original]. But if the logical probability of "All swans are white or black" is equal to the logical probability of "All swans are white"

[9] It is also possible that Popper was defending a somewhat weaker theory than ($):

(¥) Degree of empirical content of *h* relative to *b* is < / = / > degree of empirical content of *h** relative to *b** iff degree of logical content of *h* relative to *b* is < / = / > degree of logical content of *h** relative to *b**.

Thanks to Tim Williamson for drawing this to my attention.

(relative to b), then the degree of empirical content of each is equal (relative to b) on such a view. And each is a universal hypothesis, so each does have the same logical probability (relative to b), namely zero, on the argument which led us to reject (1). Hence (£) is false if degree of empirical content is to be thought of in terms of potential falsifiers. (The potential falsifiers for "All swans are white or black" are a proper subset of the potential falsifiers for "All swans are white".) Derivatively, ($) is also false if degree of logical content is defined as $1 - \mathrm{P}(h|b)$.[10]

One could resist this conclusion only by insisting that degree of empirical content is a coarse-grained measure, such that degree of empirical content of h may be equal to degree of empirical content of $h*$ even when the potential falsifiers of one are a proper subset of the potential falsifiers of the other. But to defend (£) in such a way leads to a dilemma. It forces renunciation either of the claim that degree of testability is equivalent to degree of empirical content *or* of the claim that different universal hypotheses can have different degrees of testability (given that each universal hypothesis has the same logical probability). Popper would not have wanted to suggest that "All swans are red or orange or yellow or green or blue or violet or white or black" is generally as good (*qua* testable) a hypothesis as "All swans are white", so would not have grasped the second horn. To grasp the first, however, gives rise to the question "In virtue of what, if not greater empirical content, is 'All swans are white' more testable than 'All swans are red or orange or… white or black'?" The difficulty of arriving at a satisfactory answer strongly suggests that rejecting (£), and avoiding the dilemma altogether, would be a preferable option. Then, to repeat, ($) must be rejected too.

[10] (¥) is also false if degree of logical content is defined in this way, by the same reasoning. Let b be $b*$, and let h be "All swans are white or black" and $h*$ be "All swans are white".

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