Lingnan University
Course Syllabus

Course Title	:	Optimization
Course Code	:	CDS4008
Recommended Study Year	:	4
No. of Credits/Term	:	3
Mode of Tuition	:	Sectional Approach
Class Contact Hours	:	3 hours per week
Category in Major Prog.	:	Elective
Prerequisite(s)	:	(a) CDS1002 Calculus, and (b) SSC2113 Linear Algebra
Co-requisite	:	Nil
Exclusion	:	Nil
Exemption Requirement	:	Nil

Brief Course Description:

In a mathematical optimization problem, one seeks to minimize or maximize a real function of real or integer variables, subject to constraints on the variables. Mathematical optimization refers to the study of these problems, their mathematical properties, the development and implementation of algorithms to solve these problems and the application of these algorithms to real-world problems.

The purpose of this course is to cover the parts of calculus that are most relevant to economics and business, with an emphasis on mathematical optimization questions – finding a choice of variables that maximizes a function (e.g., utility or expected return) or minimizes a function (e.g., cost and risk) in the presence of constraints (e.g., feasibility, substainability and limited resources). This course covers foundational topics from multivariable calculus and linear algebra, topics from introductory analysis (e.g., open/closed sets, compactness, maximum theorem and implicit function theorem) and problems in unconstrained and constrained optimization (e.g., Lagrange multipliers, Kuhn-Tucker conditions and applications of convexity). Topics in linear programming and calculus of variations may also be discussed. The course will also cover the application of optimization algorithms to search for optimal and sustainable solutions for some real-world problems. Participants to the course must be aware that this is in part a course of mathematics, where formal statements (definitions, lemmas, propositions, theorems) will be made and formal proofs will be given and illustrated by examples.

Aims:

The general objectives of the course are to:
1. Introduce the fundamental concepts of Optimization Techniques;
2. Learn to solve optimization problems in realistic scenarios;
3. Provide the concepts of various classical and modern methods for constrained and unconstrained problems, both single variable and multivariable.

Learning Outcomes (LOs):

Upon successful completion of this course, students will be able to:
1. Explain the basic concepts and mathematical formalism of mathematical optimization.
2. Identify, formulate and solve optimization problems with optimization algorithms;
3. Apply the concept of optimality criteria for various type of optimization problems;
4. Solve various constrained and unconstrained problems in single variable as well as multivariable;
5. Identify the right formalization and algorithm to solve real world optimization problems.

Indicative Contents:

- One-variable optimization
- Linear algebra: linear independence, rank, linear systems
- Linear algebra: Gaussian elimination, eigenvalues and diagonalization
- Spectral theorem and quadratic forms; positive/negative definiteness.
- Multivariable calculus: partial derivatives, gradients, unconstrained optimization
- Unconstrained optimization continued; second-order criteria; Convex sets
- Convex Functions and optimization
- Topology: open sets, Closed sets, bounded sets, and sequential compactness
- The maximum (Weierstrass) theorem
- Implicit function theorem
- Lagrange multipliers - second order criteria; bordered Hessians
- Convex duality
- Inequality and mixed constraints; Kuhn-Tucker conditions
- Convex optimization with constraints: Slater conditions; sufficiency criteria
- Linear programming (LP)
- Classic LP problems: optimal assignment, resource allocation, maximal flow problems
- Linear programming duality.
- The simplex and revised simplex algorithms.

Teaching Method:

The concepts and principles of optimization algorithms will be discussed during lectures with theories explained in the context of examples. Mathematical definitions and theorems will be given and proved. The instructor will provide in-class examples while students need to work on in-class of after-class exercises to solve simple mathematical variants of mathematical properties hinted during the course.
Several implementations of optimization algorithms will be demonstrated during the practical sections, so that students can realize how to identify, formulate and solve optimization problems using implementations in Python, with the help of large language models to effortless, but controlled, programming. Students will be required to perform case studies that require an understanding of relevant concepts, principles and techniques of solving real world optimization problems.

Measurement of Learning Outcomes:

1. Class Attendance and Participation: In class, students need to understand and recognize the mathematical methods for solving optimization problems. Attendance will be taken in each lecture and grades also depend on whether students actively participate in discussion and offer constructive views. [For learning outcomes 1-2.]
2. Assignments: Students undertake after-class assignments that require an understanding of relevant mathematical concepts, principles and techniques of solving optimization problems. [For learning outcomes 1-5.]
3. Mid-term test and final examination: These two closed-book assessments are designed to see how far students have achieved their intended learning outcomes. Questions will primarily be skills and understanding based to assess the student's versatility in mathematical optimization. [For learning outcomes 1-5.]

Assessment:

Class Participation	10%
Assignments	20%
Midterm Test	30%
Examination	40%
Total	100%

Required/Essential Readings:

1. Hands-on Mathematical Optimization with Python, Krzysztof Postek, Alessandro Zocca, Joaquim A.S. Gromicho, Jeffrey C. Kantor, Cambridge University Press, 2025
2. Introduction to Nonlinear Optimization: Theory, Algorithms, and Applications with Python and MATLAB, Second Edition by Amir Beck, MOS-SIAM Series on Optimization 2023
3. Knut Sydsaeter, Peter Hammond, Arne Strom, and A. Carvajal, *Essential Mathematics for Economic Analysis, 5th Edition*, Pearson, 2016.
4. Henry C. Pinkham, *Analysis, Convexity, and Optimization*, (http://math.columbia.edu/~pinkham/Optimizationbook.pdf)

Recommended/Supplementary Readings:

1. Pyomo Optimization Modeling in Python, Michael L. Bynum, Gabriel A. Hackebeil, William E. Hart, Carl D. Laird, Bethany L. Nicholson, John D. Siirola, Jean-Paul Watson, David L. Woodruff, Springer 2021
2. An Introduction to Optimization: With Applications to Machine Learning by Edwin K. P. Chong Wiley, Fifth edition, 2025
3. Knut Sydsaeter, Peter Hammond, Atle Seierstad, and Arne Strom, *Further Mathematics for Economic Analysis, 2nd Edition*, Pearson, 2008.
4. Introduction to Operations Research: 2024 Release ISE by Frederick Hillier, Mc Graw Hill 2024

Important Notes:

(1) Students are expected to spend a total of 9 hours (i.e. 3 hours of class contact and 6 hours of personal study) per week to achieve the course learning outcomes.

(2) Students shall be aware of the University regulations about dishonest practice in course work, tests and examinations, and the possible consequences as stipulated in the Regulations Governing University Examinations and Coursework. In particular,

plagiarism, being a kind of dishonest practice, is "the presentation of another person's work without proper acknowledgement of the source, including exact phrases, or summarised ideas, or even footnotes/citations, whether protected by copyright or not, as the student's own work". Students are required to strictly follow university regulations governing academic integrity and honesty.

(3) Students are required to submit writing assignment(s) using Turnitin.

(4) To enhance students' understanding of plagiarism, a mini-course "Online Tutorial on Plagiarism Awareness" is available on https://pla.ln.edu.hk/.

(5) Students must adhere to the University's guidelines and practices when using Generative Artificial Intelligence (GAI) tools. The official documents *"Guidelines for Using GAI Tools at Lingnan University" and "Best Practices for Ethical and Responsible Use of GAI Tools in Course Assessments" can be found here: https://www.ln.edu.hk/tlc/generative-artificial-intelligence/gai-guidelines-best-practices* .

Rubric for In-Class Participation of CDS4008 – Optimization

Criteria	Excellent	Good	Average	Below Average	Poor
Explain the concepts of mathematical optimization	Accurate grasp of the concepts, ability to prove mathematical results.	Good grasps of the concepts. Ability to reproduce the proof of the main mathematical results	Some grasp of the concepts, understand the mathematical concepts and proofs.	Little grasp of the concepts	Fails to grasp the concepts
Identify, formulate and solve optimization problems with optimization algorithms	Able to identify, formulate and solve most optimization problems with optimization algorithms	Able to identify, formulate and solve some optimization problems with optimization algorithms	Able to identify, formulate and solve basic optimization problems with optimization algorithms	Only able to identify, formulate and solve limited optimization problems with optimization algorithms	Fails to identify, formulate and solve optimization problems with optimization algorithms

Rubric for Assignments, Midterm Test and Final Examination of CDS4008 – Optimization

Criteria	Excellent	Good	Average	Below Average	Poor
Explain the concepts of mathematical optimization	Accurate grasp of the concepts and mathematical proofs.	Good grasp of the concepts and mathematical proofs.	Some grasp of the concepts and mathematical proofs.	Little grasp of the concepts and mathematical proof	Fails to grasp the concepts and mathematical proofs.
Identify, formulate and solve optimization problems with optimization algorithms	Able to identify, formulate and solve most optimization problems with optimization algorithms	Able to identify, formulate and solve some optimization problems with optimization algorithms	Able to identify, ormulate and solve basic optimization problems with optimization algorithms	Only able to identify, ormulate and solve limited optimization problems with optimization algorithms	Fails to identify, ormulate and solve optimization problems with optimization algorithms
Apply the concept of optimality criteria for various type of optimization problems	Able to apply the concept of optimality criteria for most optimization problems	Able to apply the concept of optimality criteria for some optimization problems	Able to apply the concept of optimality criteria for basic optimization problems	Only able to apply the concept of optimality criteria for limited optimization problems	Fails to apply the concept of optimality criteria for optimization problems
Solve various constrained and unconstrained problems in single variable as well as multivariable	Able to solve most constrained and unconstrained problems, ingle variable as well as multivariable	Able to solve major constrained and unconstrained problems, single variable as well as multivariable	Able to solve basic constrained and unconstrained problems, single variable as well as multivariable	Only able to solve limited constrained and unconstrained problems, single variable as well as multivariable	Fails to solve constrained and unconstrained problems, single variable as well as multivariable
Apply optimization algorithms to real world problems	Able to apply most of the optimization algorithms to real world problems	Able to apply some optimization algorithms to real world problems	Able to apply limited optimization algorithms to real world problems	Only able to apply the basic optimization algorithms to real world problems	Fails to apply optimization algorithms to real world problems